AMERICAN SOCIETY OF CIVIL ENGINEERS.

INSTITUTED 1852.

TRANSACTIONS.

336.

(Vol. XV.—July, 1886.)

ADDRESS

At the Annual Convention, at Denver, Colo., July 2d, 1886,

BY

Henry Flad, President Am. Soc. C. E.

The field of engineering science has of late years become so extensive, that the task imposed on your President of giving a summary of engineering progress during the year is one of considerable magnitude. To make such a summary exhaustive, the records of all civilized countries would have to be searched, and the merits of new methods and appliances carefully weighed, so that only those which have proved, or given promise, of real value may be brought to your notice.

It would be impracticable within the limits of this address to even enumerate all the important engineering works commenced or completed during the year; and to present simply a statement of the dimensions and cost of such works would be of slight interest or benefit to the Members of the Society. I have therefore concluded it would be preferable that I should mention some of the most important works only, and devote a portion of this address to a statement of my individual views on

such other matters as I judge to be for the well-being and progress of the profession. Even if some of my views should be shared by but few of those to whom I speak, no harm can ensue, as—very properly—the Society is not responsible for the individual opinions of its Members. Furthermore, a departure from precedent to this extent, and from what may be termed the strict construction of the By-Laws, will doubtless lead to more general consideration of the subjects to which I refer.

There is another point on which I propose to depart from the beaten path. Instead of giving the greatest prominence to those works of the engineer which never fail to excite general admiration, by being either the largest or the most difficult, or perhaps the most expensive, I propose to give the first consideration to that part of the work of the engineer which more directly influences the health and comfort of the human race.

I will therefore commence by referring to municipal engineering, by which I mean those branches of engineering which provide abundant and pure water, good streets, and safe buildings, and which keep pestilence from our dwellings by providing for the prompt removal of all offal and *debris*, or by applying proper means for destroying their power to work harm.

A liberal supply of wholesome water is an essential condition for health and comfort. This has been recognized most fully in our own country. Even the smallest towns are erecting water-works, and the large cities are either extending their existing works, or are engaged in investigating the best method for improving, as well as for increasing their supply of water.

When the planning and construction of works of this kind is entrusted to experienced and skillful engineers, as is generally the case in our large cities, and when the usefulness of the engineer is not impaired by the interference of politicians, or of another class of men who may be called amateur engineers, the works are substantially constructed with a view to economical operation, and with due regard to the necessities of the future.

In the smaller cities or towns the mode generally adopted is for one of the many water-work companies or syndicates to undertake the planning and construction of water-works under a contract by which they guarantee a number of fire streams of a fixed height at a certain price

per annum, and also to supply water to the inhabitants at a rate agreed upon. But as the engineer in charge of the municipal work too often has no special knowledge of hydraulic engineering, it is but natural that in most cases the lowest bid will be accepted, and just as natural that the company or syndicate which obtains the franchise for the construction and operation of the works should plan them with a view mainly to cheapness in first cost, and without any regard either to the quality of the water to be furnished, or to the permanency of the works. That under such a system many of the water-works of our smaller cities should be more or less failures, is hardly to be wondered at.

There are a number of companies in this business which have a reputation to uphold, and employ competent engineers to plan and to construct the works for which they contract; but unless the city authorities should possess greater wisdom and knowledge of hydraulic works than they are ordinarily gifted with, some speculating syndicate will underbid, as it safely may, the responsible firm or company, and by building poor works inflict a lasting injury on the community.

Of new devices used in operating water-works, I may call attention to the æration of water, by means of air introduced into the pipes under pressure, for the purpose of destroying the organic germs which otherwise might affect the salubrity or taste of the water.

A new method of investigating the purity of potable water has also been introduced. It consists in observing the colonies of microbes which are produced in gelatine that has previously been sterilized. This method was first used in Berlin, and probably leads to more correct conclusions in regard to the salubrity of the water than any chemical analysis.

As regards cleaning the water from solid matter carried in suspension, our country cannot claim to have reached the high standard that obtains in other countries. American practice, in cases where the water is taken from turbid streams, generally provides only for removing so much of the material held in suspension as will subside during a certain period of quiescence in settling basins.

Filtration, so largely used in England and on the Continent, is used here in but few instances. One reason why it is not more generally adopted, probably lies in the fact that the quantity of water consumed *per capita* in this country is largely in excess of all legitimate requirements. Of late years the quantity *per capita* has been reduced by

various means adopted to prevent waste. If efforts in this direction are continued, and are reasonably successful, measures for improving the quality of our potable water will probably be more generally adopted.

The most important and extensive works for the supply of water which are being carried on at present, are those of New York in the United States and Liverpool in England. The New York works are intended to provide an ultimate daily supply of 320 000 000 gallons. The main features are a masonry dam, 178 feet above the bed of the stream and 1 300 feet in length, forming a lake, which will contain 3 200 000 000 gallons. As the masonry of the dam must be sunk more than 100 feet to reach solid rock foundation, its total height will be nearly 300 feet, and its width at the bottom about 200 feet. The aqueduct which is to carry the water to the Central Park Reservoir is to be 31 miles in length, and of an average diameter of 14 feet. It will cross the Harlem River by an inverted syphon, 150 feet below tide level. The construction of the aqueduct is rapidly progressing.

Of similar character, and of hardly less magnitude, are the works for the supply of water to Liverpool. The Vrynwy Dam, of Cyclopean masonry, is to be 136 feet high in the center, 1 258 feet in length, and 117 feet thick in its widest part, and the impounded water will cover 1 118 acres. The aqueduct will be 35 miles in length.

The work of the engineer next in importance, if its effect on health be made the criterion, consists in providing for the removal of all refuse matter which if allowed to remain would affect the health or comfort of the community. The need of works of this kind has become so fully recognized of late years, that their construction has grown into a special branch of engineering.

The removal and disposal of human *excreta*, and of the waste water from dwellings and factories, present greater difficulties than that of any other kind of refuse. Both scientific attainments and experience are required in the engineer who undertakes to select that system of sewerage which is best adapted to local conditions and requirements. The system of carriage by water is evidently the only one practicable in a country where the water-closet is so generally used as in the United States.

Whether sewage proper should be taken into the conduits which, in most cities, it is necessary to construct to carry off storm water, or

whether it should be carried off separately—in other words, whether a combined or separate system of sewerage should, for any particular city or town, be chosen—cannot be properly determined without thoroughly weighing the advantages and disadvantages of either system, as applied to the particular place requiring the improvement. To insist that one of these systems should be exclusively used in all cases is, in my opinion, a position which the engineer ought not to assume.

With the combined system, the only rational method for the final disposal of sewage consists in its discharge into large bodies of water in motion, except, perhaps, where extensive tracts of sandy soil are available.

Instead of entering into a discussion of the different methods used for finally disposing of the sewage under the separate system, I will state that, in my judgment, the filtration of sewage, by allowing it to flow over large areas of under-drained land (as is practiced with perfect success in many towns in England, and which has also been introduced at the City of Pullman, Ill.) is the most certain and most generally applicable method.

The appliances for receiving the sewage and conducting it from the houses—or what may be designated as plumbing fixtures—have lately been receiving much more attention than formerly, and our people are slowly learning, some of them by dire experience, that in building dwellings they should look to perfect design and workmanship in the execution of plumbing work, rather than to its cheapness, if they desire to secure healthy homes.

Speaking of the disposal of refuse matter, I will mention that the burning of street sweepings and offal, in ovens constructed for the purpose, is successfully and economically carried on at Leeds and other places in England, and express the hope that the example will soon be followed in our own country. But the burning of street sweepings can be economically carried out only when the pavements are constructed of material which is not readily abraded by the traffic.

The necessity of procuring pavements of this kind is fully recognized, and the streets of Paris, London, Berlin, Vienna, New York, Boston, Chicago, St. Louis, Baltimore, Cincinnati and Washington, are now nearly all being paved with the same kinds of material, and under almost identical specifications. Granite blocks, wooden blocks and

monolithic asphaltum are almost exclusively used in the principal streets of these cities. The granite pavement is the most durable and requires the least repairs; the monolithic asphaltum takes precedence from a sanitary point of view; and the pavement made of blocks of wood, as lately constructed, is preferred by some on account of its being easiest on horses and least noisy. But it still remains for the engineer to devise a pavement, durable and smooth, for use on streets where the traffic is not heavy enough to require granite, or where the cost of either of the two other pavements above mentioned is too great to justify their application.

Macadam pavements, when good stone can be obtained, will, under proper treatment, give a smooth surface, agreeable for travel, provided they are kept in first-class order by the immediate repair of any defect. But this is a condition which can hardly be realized in this country on account of the high price of labor.

Street pavements of hard-burnt brick are now being tried on a large scale in a number of towns in Illinois. At Bloomington such a pavement has been in use since 1876. Blocks composed of coal tar, pitch and sand, and called ceramite, have been used for street pavements at Buda-Pesth since 1881. Both these pavements are laid in a similar manner.

A street pavement of round cedar blocks has been largely introduced in Western cities of late years, but can hardly be considered to fill the conditions required for a good pavement, on account of the unequal wear of the sap and heart wood.

Among the things which affect the health and comfort of the inhabitants of cities using bituminous coal, is the smoke from factories and dwellings. Many appliances for smoke prevention have been devised, some of which are reasonably successful. Their application mitigates the evil, if it does not wholly remove it; but it is to be hoped that the experience gained at points where natural gas has been found and applied to heating purposes, and the late improvements in the manufacture of heating gas, will before long lead our people to abandon the primitive mode of heating with solid fuel now in vogue, and thus abate the smoke nuisance.

A new problem is pressing itself upon the attention of the municipal engineer—one that presents special difficulties, and will require his best

endeavors to satisfactorily solve. This is the proper method of arranging and carrying the numerous wires required for the electrical appliances which are now used in our cities and towns. In the larger cities, where space is limited and the purposes to which electricity is applied are various, necessitating the employment of all kinds of electrical currents, many of which act injuriously upon each other, the electrical difficulties to be overcome are great, and by many are considered to be of such magnitude as to forbid the use of routes or ways in common. In the larger cities the public demand that the wires shall be placed underground, and undoubtedly they must be so placed to a greater or less extent.

The electrical difficulties need not be considered here. The engineering difficulties are in themselves of sufficient importance. The solution of the problem which most naturally suggests itself is the building of large subways, in which all the wires may be placed, as well as the water and other pipes, pneumatic tubes and other modern appliances; the subways to be of a size admitting men to enter at convenient points for placing and repairing the wires and pipes without interfering with the street traffic. The cost of such subways would be so great as to forbid their construction except to a limited extent.

On the other hand, the use of small conduits or tubes, into which the wires shall be drawn through numerous openings in the street surface, will necessitate almost constant interference with traffic, and frequent disturbance of the pavement of the streets and sidewalks for enlargements and repairs, and for making connection with buildings.

I am inclined to think that what may be designated as the mixed system would be the most satisfactory, under which large subways would be used on streets where the number of electric conductors and of pipe lines required for all purposes was sufficient to warrant the expenditures for such structures, while for the other streets a system of ærial carriage and distribution might be devised which would be free from the gravest objections to the present practice, and which would admit of gradual expansion without interrupting traffic until the proper time for the construction of a subway had arrived.

Before dismissing the subject of municipal engineering, I desire to say a few words in regard to the difficulties which are in the way of a proper management of the engineer department of our cities and the

means of removing them; not specially for the benefit of the profession, but of the community at large. The greatest obstacle to the proper conduct of public work in cities and villages arises from the fact that the engineer is frequently selected from political preferences, which often gives the position to an incompetent man. Another obstacle is that the term of office is generally limited by law to one or two years, which does not allow enough time for even a competent engineer to become thoroughly acquainted with his duties, and with the distinctive features and requirements of the locality, before he is liable to be displaced. If the evils arising from such a state of affairs are to be avoided, appointments should be based on strict civil service principles. No person should be appointed on any other ground than that of competency and honesty, and he should hold his office during good behavior; and all subordinates should be selected on the same principles.

Some of our large cities, recognizing the evil effects of political appointments of officers in charge of public works, and of frequent changes, have tried to remedy these by placing the general government in the hands of boards of commissioners, generally selected from among prominent business men in the community. Changes of this kind have usually been made when a reform wind was blowing, it being supposed that, although changes in the members of these boards might take place, the intelligence of such a class of men would be a guarantee against causeless changes of employees. This system, while it has been an improvement on the old one, has the disadvantage that members of these boards are too often hardly more than figure-heads, whose duties might just as well be performed by the engineers entrusted with the planning and execution of the public works. To make a board of public works thoroughly useful, the majority of the members should be engineers, each of whom should take charge of a particular department as its chief. By this system the board would have the benefit of the special information of each member as regards his own department, as well as of his general knowledge of engineering matters. St. Louis, Providence and Milwaukee have partially adopted this system, and it is to be hoped that other cities will follow their example.

In connection with this subject, I am pleased to record the fact that in some cities positions on public works are now made dependent on the result of examinations held under civil service rules.

Next in importance to municipal engineering, I consider the work of the railroad engineer.

During 1885 about 3 000 miles of railroad were constructed in the United States. This is less than for any one year since 1878; yet there seems now to be some revival in railroad construction, as from January 1st to June 1st, 1886, 1 100 miles of railroad have been constructed, as against 600 miles during the corresponding period of 1885.

The total mileage of railways in the United States may now be taken as about 129 500 miles.

The rapid increase of railroad traffic, and the introduction of freight cars carrying as much as 60 000 pounds, has directed the attention of railroad managers to the necessity for the improvement of brakes for freight trains.

In a paper read before our Society, it has been stated that the number of freight cars in the United States provided with power brakes on June 1st, 1885, probably did not exceed 4 per cent. of the total number. In June, 1885, the Master Car Builders' Association instituted a series of experiments, with a view of determining the merits of different brakes proposed for freight service. Another set of experiments upon automatic brakes is to be made under the auspices of the same Association, at Burlington, Ia., on July 13th, 1886, in which the best brakes now in use will be brought into competition. The results of such competitive tests will be of great value to railroad interests.

Of the various devices used for increasing the safety of railroad operations, one of the most effective and important is the block signal system, almost universally used in England, and which it is to be hoped will before long find general application in our own country.

A great obstacle to improvement in railroad transportation arises from the great variety of designs prevailing in the rolling stock and other appurtenances of railroads, and it would contribute greatly to safety and economy if uniformity in these respects could be attained.

If all, or a considerable number of railroad companies would combine to establish, at common expense, a station where experiments and tests as to the value of railroad plant, rolling stock, and appliances could be made on a large scale, and under the direction of scientific and practical men, it would save them the expense of individual efforts, and secure much more trustworthy results.

As an instance in which the railroad companies have perceived the

benefits arising from uniformity, I may mention the change of gauge from 5 feet to the standard, lately made on the Southern roads. Between May 31st and June 3d of this year the gauge on 11 500 miles of road was changed with perfect success, and with hardly any interruption of traffic.

A great advance in the construction of mountain roads has been made by Abt, whose system has been successfully applied on the Hartz Railway.

Ship railways have heretofore been used for transporting small vessels, but it was reserved for a Member of this Society to work out the details of the problem of applying railroads to the transportation of the largest sea-going vessels long distances overland in a manner which is fully indorsed by a number of the most eminent engineers and naval constructors.

The construction of the Chignecto Ship Railroad from the Gulf of St. Lawrence to the Bay of Fundy, similar in its details to those proposed for Tehuantepec, is now under contract, and the project of a ship railroad across the Peninsula of Florida is being discussed.

The system of elevated railways is remarkably successful in the City of New York, and is being extended to the suburban districts north of the city. There are also similar lines in operation, or in progress of construction, in Brooklyn and Kansas City.

Cable tramways were first used in San Francisco in the year 1873; in New Zealand, 1880; Chicago, 1882; London, England, 1884; Melbourne, Philadelphia, and Kansas City, in 1885; and during the present year in New York and St. Louis. For short lines this system has many advantages, but for long lines the use of some other motor will probably be found more economical.

Electricity has for several years been in use for operating city and suburban tramways. There are different methods of carrying the electric current from the central station to the cars. In almost every case a separate conductor is used, either overground, as at Berlin, Baltimore, and the Giant's Causeway, or in slotted conduits similar to those used on cable tramways, but of smaller dimensions, as at Cleveland and on the Blackpool line. Electric tramways have been tested during the year, or are now in process of construction, at Toronto, Can.; New Orleans, La.; Minneapolis, Minn.; Detroit, Mich.; Montgomery, Ala.; Denver, Colo.; and Appleton, Wis.

Different from these systems is that of Reckenzaun, who carries the electricity for operating the motor along with the car in a storage battery. This system is now being tried at Battersea, England. If storage batteries can be constructed which, besides being light and durable, will return a fair percentage of the power used in charging them, this system would give promise of great economy and convenience.

The rapid development of the application of electricity to many of the necessities and comforts of civilized life, would seem to demand that more attention than heretofore should, by the profession at large, be paid to this branch of engineering. A knowledge of this science will within a few years be just as necessary to the civil engineer as is to-day the knowledge of hydraulics.

The building of inland canals has on this continent nearly ceased, as railroad transportation, although more expensive, insures greater regularity and dispatch, and is not subject to stoppage during winter as are our Northern canals. Canal work is now almost entirely confined to deepening and widening the existing lines. The Welland and the Erie Canals are being thus treated. The locks of the Sault St. Marie Canal have been deepened so as to admit vessels of 10 feet draught.

But while the construction of inland canals is now at a standstill, a number of ship canals are in progress or projected. The most important of these, the Panama Canal, is, according to latest advices, not making such progress as to satisfy the friends of the enterprise. Indeed it would appear that, if completed at all, it may have to be changed into a canal with locks.

The canal across the Isthmus of Corinth is in progress.

Among projects for ship canals, which are reasonably certain of being carried out in the near future, is the canal between the Baltic and the North Sea, and the Manchester Canal in England. Canals are also proposed from the Baltic to the White Sea *via* Lake Onega, and across the Isthmus of Nicaragua.

These engineering works, if completed, cannot fail to exercise a very beneficial influence on the commerce of the world.

The subject of irrigation is of the utmost importance to some of our States and Territories, and a large amount of this class of engineer-

ing work has been done in California, Arizona, Colorado, Dakota, Montana, Idaho, Oregon and Georgia. . The total length of ditches, pipes and flumes is estimated at over 10 000 miles. The greatest progress in regulating the collection and distribution of water for purposes of irrigation has been made in Colorado, and the wise laws enacted by that State are considered and used as models by other States.

From the excellent report of the State Engineer of Colorado, it appears that there are over twenty-six water districts in existence in the State, distributing over 1 800 000 cubic feet of water per minute for purposes of irrigation. The number of acres irrigated in this State exceeds 1 400 000. An irrigating canal, 30 miles in length, is now under construction at Catlin, Colorado.

The extent and cost of some irrigating works is shown by the canal in San Joaquin Valley in Merced County, California. This canal is about half completed, and will, when finished, have a total length of 35 miles. It will pass through two tunnels, one 1 600 feet, the other 6 000 feet in length. The total cost of the work is expected to be $1 500 000.

A still larger irrigating canal is being excavated in Northern Wyoming, and will be more than 100 miles in length.

The improvement of our rivers, and more particularly of the Mississippi River, in which one-third of the population of the United States is directly interested, has made but little progress during the past year, owing to the failure of Congress to make the necessary appropriations. A costly plant of steamboats, barges, pile-drivers, etc., has been prepared for the prosecution of this work. Not only does all this plant remain unused, but the works already begun, which avowedly had been constructed rather light for the purpose they were intended to serve, are gradually giving way under the attacks of the river, and before long, if not cared for, will altogether disappear. This making appropriations sufficient to start work one year, and refusing next year even the money necessary to secure and complete what has already been commenced, is certainly the height of folly, and it is to be hoped that Congress, when arriving at a full understanding of the effects of such action, will not hesitate to change it. The completion of the Davis Island Dam on the Ohio River during the year marks a very decided forward step in the improvement of that river.

The storage reservoirs which have been constructed on the Missis-

sippi River, 300 to 500 miles above St. Paul, with a view to regulating the flow of water in the river, were opened on the 1st of August, 1885, and have so far fulfilled all reasonable expectations as to their effect, and it is probable that these results, achieved with a comparatively small outlay of money, will encourage the authorities to further steps in this direction.

In the field of military engineering complete success has crowned Gruson's efforts to construct shields and cupolas of chilled cast-iron. Experiments have been prosecuted in torpedo construction and service, and also in the firing of shells filled with nitro-glycerine, which are said to have been successful. In addition to the foregoing, the Maxim machine gun, operated by the recoil, is a very important recent invention in gunnery.

It is greatly to be hoped that Congress will before long see the necessity of providing for our sea-coast defenses. When this is done, the present number in the Corps of Engineers of the United States Army would hardly suffice to plan and construct these works, and they would then be fully employed in the sphere of military engineering proper, leaving the improvement of rivers and harbors, and of all works of a civil character, to civil engineers. This would be the most natural and easy solution of the question now under discussion between members of the profession as to the proper policy of conducting public works. While it is my opinion that public works of a civil character could as well be carried on by civil engineers, appointed under strict application of civil service rules specially adapted to the purpose, I am opposed to any action by our Society which might seem to be directed against the interests, or even privileges, of a number of our Members, who are second to none in scientific attainments, in experience, and in character.

Tunneling nowadays can be done much more expeditiously and cheaply than formerly, in consequence of the improvement in drills and in explosives, coupled with better methods of ventilation.

Of large tunnels recently opened to traffic, may be named those of the Severn and the Mersey.

The construction of the Cascade tunnel on the line of the Northern Pacific Railway has been commenced. It will be 9 850 feet in length, and next to the Hoosac, the longest railroad tunnel in America.

Projects are on foot for several new Alpine tunnels; among them a tunnel under Mont Blanc, to be 12 miles in length, and one under the Simplon, to be 12½ miles long. But a prediction has been made, based on the experience gained in other Alpine tunnels, that the heat in the central portion of the Simplon tunnel would prove so great as to make human life impossible.

A project for a tunnel under Northumberland Straits, between Capes Traverse and Tormantine, deserves to be mentioned, as it proposes some bold and novel features of construction, and yet seems to be well considered in all its details, and to give fair promise of success.

The project of a tunnel under the Straits of Dover, according to late information, has a fair prospect of being revived. It is very desirable that a work which will confer such great benefits on the two countries which it is to unite should be completed; particularly since the work already done has made it a certainty not only that the construction of the tunnel is practicable, but that it can be completed in a shorter time and for less money than was originally estimated.

By far the largest work of subaqueous tunneling for the purpose of removing rock impeding navigation, was that of Flood Rock in Hell Gate. The area over which the work extended comprised nine acres. By the final explosion of October 10th, 1885 (in which 225 000 pounds of racka-rock and 75 000 pounds of dynamite were used), a quantity of rock estimated at 200 000 cubic yards was shattered so as to admit of ready removal by dredging.

The greatest activity in any branch of civil engineering during the past year seems to have prevailed in bridge construction. Quite a number of important bridges have been completed, among them that across the Susquehanna River, on the Baltimore and Ohio Railroad, 6 315 feet in length, having four spans of 480 feet, and one of 520 feet; the Henderson Bridge across the Ohio River, 3 200 feet in length, with one span of 525 feet; the St. John's River Cantilever, 447 feet between piers; and the bridge across the Big Black River. Of large bridges in course of construction, the most important are the Forth Bridge, with two spans of 1 700 feet each; the Sukkur Bridge, across the Indus, having a span of 790 feet; and the Lachine Bridge, on the Canadian Pacific Railroad, with two spans of 408 feet.

The contract for the erection of a bridge at Hawkesbury, New South

Wales, has lately been awarded to one of our American Bridge Companies, a very gratifying fact when it is considered that the contract was obtained in competition with various bridge companies of England and France. The main difficulty to be overcome in the construction of this bridge lies in its deep foundations, which are to be sunk to a depth of 170 feet below the surface of the water.

The extremely low price of iron and steel greatly favors the selection of long spans for bridges, as the saving in piers and foundations balances the extra cost per lineal foot of long spans.

The tendency among bridge engineers at present seems to be favorable to the selection of systems in which the strains to which any member may be subjected can be accurately determined by calculation; and the use of the pin joint, which may be called a distinctive feature of American bridge construction, favors the attainment of this object. The rapidity with which bridges with pin joints can be erected is an immense advantage, particularly when material for such bridges has to be prepared at a great distance from its final destination, or when erection must take place where no facilities for doing iron-work exist. This system of construction is therefore particularly adapted for new and thinly settled countries.

Since the great success of the cantilever bridge at Niagara Falls, a number of other bridges have been built on this principle. Indeed by far the greater number of long span bridges lately proposed are to be cantilevers, as this system offers great advantages in erection. But they are subject to greater deflections than those built on other systems, and I believe that the arch might, in many cases, be preferable, as it gives almost the same facilities in erection, and is less deflected under the action of a moving load. I am glad to see the arch proposed in a late design for the Harlem River Bridge.

As I have before mentioned, both the weight of locomotives and of cars has greatly increased of late, and bridges constructed years ago, of sufficient strength to carry the load which at the time of their erection was considered a possible maximum, are now, by the increase in the weight of rolling stock, subjected to loads which greatly reduce their factors of safety. There are undoubtedly a great many of the older bridges which require to be strengthened or replaced if accidents are to be avoided.

I think it is the duty of the members of the profession to direct the

attention of State governments to this fact, and to suggest that thorough examination of all bridges be made. This would be a work of considerable magnitude, and would require both time and money, but in a civilized commonwealth the care for the safety of the citizens should outweigh any financial considerations.

The necessity of tests and experiments when new forms or new materials are to be introduced, will be readily admitted. To make such tests, not on small models, as at one time was considered sufficient, but on full-sized members, as has been shown to be preferable, requires large testing machines and plenty of both time and money. No individual engineer, and but few corporations, can afford to provide the necessary funds for this purpose. It was therefore proper, that for the advancement of science, as well as for the benefit of the material interests of the whole country, the United States Government should cause a testing machine of large dimensions to be built, which, under proper regulations, would be accessible to engineers and manufacturers for tests of material.

Such a machine was erected at Watertown, Mass., has been in use for several years, and has proved of very great service; yet, being under the immediate control of the Ordnance Department of the United States Army, it is, during a great portion of the time, necessarily employed in the service of that Department, and as a great many of the specimens tested are of small dimensions, the experiments made thereon are not of any value to the general public, and the primary object for which the construction of this machine was undertaken is partly defeated. A remedy would be found if the United States Government could be induced to erect, in addition to this large testing machine, some machines of smaller dimensions, and at the same time to appropriate an amount sufficient to pay for tests, not of special, but of general, interest, and for the time of engineers specially charged with the work.

While on the subject of tests, I may mention that during the past year there have been made public the results of a number of valuable experiments made by Members of this Society on cements; on the comparative value of lubricants; on evaporation; on the strength, elasticity, etc., of iron and steel; and on other matters of interest to the profession.

The use of the new system of notation of time is gradually extending,

and promises to become universal in application. The introduction of the metric system is also progressing, though not quite so rapidly as might have been expected from the progressive spirit of our nation. It seems strange that in this age of rapid interchange of goods and thoughts between civilized nations, such an obstacle as the use of different measures in different parts of the globe should be allowed to exist. Nobody can doubt that this obstacle will be removed before long, and the only question can be, what system should be adopted.

There are now 242 000 000 of people using the metric system, and the weight of numbers is probably already on the side of that system. This may be balanced, or even outweighed, by the industrial prominence of the nations which use the English standard; but the selection should clearly not be made either on the basis of the greater number now using a particular system, nor on the cost of the change in money, or in temporary inconvenience, but it should be made on the intrinsic merits of the system. And there can be no doubt that the metric system fulfills almost every condition of a perfect system of measurement, and could hardly be improved.

Among the strong objections to the introduction of the metric system, has been the necessity which it involves of a partial remodeling of the tools in our workshops. This would undoubtedly be very serious, but the saving in time to all classes who are engaged in any kind of business requiring measurement and calculation (and there are but few which do not require them) would soon make up for this loss. As to the inconvenience resulting from its introduction in the ordinary walks of life, it will, judging from the experience of Germany, hardly be felt in a country in which education is so universally distributed and the mental activity of the people so strongly developed as in our own. But even our manufacturers may be willing to agree to the change when they consider that many countries showing rapid progress, such as Brazil, Mexico and the Republics of South America, have adopted the metric system, and that this gives to France and Germany a great advantage over the United States in selling to these countries their manufactures.

I am glad to be able to state that a bill has been introduced in Congress which prescribes that, after the 4th day of March, 1892, the metric system shall be exclusively used in all transactions in which the Federal Government is concerned.

I may congratulate you upon the rapid growth of the Society, both in numbers and influence, but it is yet far behind the Institution of Civil Engineers of England as to numbers, and probably even more so as regards influence. This is mainly due to the fact that English capital has been largely employed in the English colonies, and in foreign countries, in enterprises managed by English engineers; while our capital and the services of our own engineers were, until lately, almost exclusively needed at home; and to the further fact that the English people hold the profession which produced some of their greatest men, such as Watt, Smeaton and Stephenson, in higher esteem than has been the wont of our own fellow countrymen.

But our country is getting both richer and wiser every day, and I hope the time is near at hand when American engineers will attain a wider sphere of action and a higher degree of public esteem. Those of us who have seen engineering, as a profession, start into life in the United States, may not see that hope realized, but we have the satisfaction of knowing that we have earnestly striven for the attainment of this end.

AMERICAN SOCIETY OF CIVIL ENGINEERS.

INSTITUTED 1852.

TRANSACTIONS.

NOTE.—This Society is not responsible, as a body, for the facts and opinions advanced in any of its publications.

337.

(Vol. XV.—July, 1886.)

ON THE STRENGTH OF COLUMNS: DISCUSSING THE EXPERIMENTS WHICH HAVE BEEN ACCUMULATED, AND PROPOSING NEW FORMULAS.

By THOMAS H. JOHNSON, M. Am. Soc. C. E.

READ AT THE ANNUAL CONVENTION, JUNE 26TH, 1885.

The history of the efforts made in the past fifty years to ascertain the law governing the strength of long columns is well known to the profession; nevertheless a brief *résumé* of the efforts made in this direction will not be out of place at this time.

In nearly all other forms in which the materials of construction are used, the relations between force and resistance can be readily deduced by analytical methods; but in the case of columns, that method of investigation has not proved satisfactory. Euler gave us a formula for the flexure of very long columns, obtained from the equation of the elastic line, but this did not meet the wants of the engineer for the cases arising in ordinary practice, and we have been compelled to resort to empirical methods of solving the problem.

Passing over the earlier experiments of Tredgold, Barlow, Rondelet and others on the strength of wood, which, because of its cheapness, made a rigid adherence to exact proportions less imperative, we come to the period, about 1830 to 1840, when iron began to attain greater prominence as a building material. The higher price of this material made it important that engineers should deal more exactly with the problem, so that while building safely, they might not build wastefully.

The well-known experiments made by Mr. Eaton Hodgkinson in connection with the building of the Conway and Britannia Bridges, have been made the basis of all the formulas that have been proposed, the methods of obtaining the formulas being in all cases purely empirical. Mr. Hodgkinson himself was unable to obtain a *single* formula that would agree with the whole series of experiments throughout all length ratios. He established certain limits, and gave to the profession two separate formulas. And further, he found it necessary to fix the limit differently as applied to square and round-end columns. His formulas served a very useful purpose, although they were crude in form and difficult of application.

Mr. Gordon, taking the same series of experiments, and proceeding on the assumption that a column to support a given load requires a certain area to resist the direct compression, plus a certain additional area to resist the tendency to flexure, succeeded in obtaining a formula that would meet, reasonably well, the whole series of experiments. It was

$$P = \frac{f A}{1 + a \frac{l^2}{h^2}}$$

in which

$P =$ ultimate load.

$A =$ area of the section.

$f =$ modulus of compression of the material.

$l =$ length of column.

$h =$ least dimension of the section.

$a =$ a constant.

The a, as deduced from the experiments, was a compound function of the form of cross-section and the bending resistance of the column.

Prof. Rankine separated from Gordon's a that part due to the form of cross-section, by substituting for the least side of the rectangle the corresponding radius of gyration. But his a was some unknown func-

tion of the resistance to flexure, and the formula could not be applied to other materials than those embraced in the experiments (cast and wrought-iron) without other extended experiments.

The simplest forms in which force can be applied to a body are by direct tension or compression. The amount of stretching or shortening which the material undergoes is determined by the modulus of elasticity. The capacity of a material for resistance in these three respects may be called elementary moduli. Resistance to force in all other modes of application must be combinations or modifications of these. The relation which Rankine's a bore to these elementary moduli was not determined, and hence the necessity for special experiments to enable us to apply the formula to other materials.

In 1872 the Vienna *Bauzeitung* published a paper by Mr. E. Hatzel, of Bavaria, in which the author started out with the same assumption made by Gordon, that the work done by the imposed load is two-fold, viz.: direct compression, together with bending of the column; and he succeeded in establishing, analytically, a formula which is identical with Rankine's, except that he goes a step further, and determines the relation which Rankine's constant bears to the elementary moduli of the material, it being

$$a = \frac{K}{\pi^2 E}$$

When empirical and theoretical methods lead to the same result, it would seem that the evidence of the correctness of the formulas so established was complete and satisfactory. But engineers have gradually come to distrust all the formulas, and to believe either that the experiments have not been properly interpreted, or that iron has one mode of behavior in small test-pieces, and another and different one in full-sized members as used in structures. To this feeling of doubt and uncertainty we are indebted for the large number of experiments upon full-sized columns which have been made in recent years, and which form the basis of this paper.

While studying the diagrams which accompany the interesting papers on the "Strength of Iron and Steel," in the Transactions of this Society for April and August, 1884, by Mr. James Christie, M. Am. Soc. C. E., the present writer was impressed not only with the great want of conformity of the diagrams of his experiments on struts, to the curves corresponding to any of the formulas hereto-

fore in use, but also with the marked irregularity which seemed to defy all law. Believing that this irregularity was due solely to an insufficient number of experiments, so that the whole field was not fully occupied, the published results of all experiments on columns which were accessible to the writer were collated and plotted, in the hope that the different groups of experiments when thus brought together would so far supplement each other as to fill out the irregularities, and, by fixing well-defined limits to the field covered by the plotted experiments, indicate the central law governing them.

The experiments available and used, include, besides Mr. Christie's, the Cincinnati Southern; the Watertown; Mr. Hodgkinson's, both cast and wrought-iron, and such others not included in the above as could be culled from various sources. In plotting them no attempt was made to distinguish the different authorities, but the different classes of end-bearings were plotted on separate sheets. In the case of flat ends, the field of experiment proved to be sufficiently extended to indicate the law very clearly, and in the cases of other end-bearings, and of the flat-ended mild and hard steel, the conformity to this law is very plainly seen when the law is once known. No regard was paid to arithmetical averages, it being deemed safer to draw a line bisecting, as nearly as possible, the general field. The lines so drawn for the different materials and end-bearings were then carefully compared, to ascertain, if possible, their mutual relations.

In making this comparison, the following points were ascertained:

First.—That part of the line corresponding to the higher length ratios is a curve, the equation of which is Euler's formula given below.

Second.—That part of the line corresponding to the lower length ratios is a straight line tangent to the aforesaid curve, and intersecting the vertical axis at a point which is constant for each material in all the varieties of end-bearings.

Third.—The curve of Euler's equation possesses a peculiar property not heretofore noticed so far as the writer is aware, viz.: that for all tangents to the curve the ordinate at the point of tangency is one-third of that part of the vertical axis intercepted by the tangent.

Euler's formula is usually written

$$P = \frac{E a I}{l^2},$$

in which

$P =$ ultimate load supported by the column.

E = modulus of elasticity of the material.

I = moment of inertia of the cross-section.

l = length of column.

a = a constant.

For our purpose it will be more convenient to substitute for I its equivalent, $A\,r^2$, in which

A = area of cross-section.

r = radius of gyration of cross-section.

The equation then becomes

$$P = \frac{E\,a\,A}{\left(\frac{l}{r}\right)^2} \qquad (1)$$

in which a is to be determined from the experiments.

The tangent line being a straight line, its equation must be of the general form

$$y = ax + b$$

in which a is the tangent of the angle which the line makes with the horizontal axis, and b is the distance from the origin at which the line crosses the vertical axis, which in our case $= K$. In our diagrams, y and x are respectively P and $\frac{l}{r}$.

In order to avoid confusion with the a in equation (1), we will use c instead of a in the equation of the straight line. Changing the other characters to conform to our notation, and observing that the line extends downwards to the right, and its tangent must therefore have the minus sign, the equation becomes

$$P = \left(K - c\frac{l}{r}\right)A \qquad (2)$$

The property, noted above, that the ordinate at the point of tangency is one-third the ordinate at zero of abscissas, enables us to compare these two equations and obtain the relative values of a and c.

Substituting $\frac{K\,A}{3}$ for P, and transposing, equation (2), for the point of tangency becomes

$$\frac{l}{r} = \frac{2\,K}{3\,c} \qquad (3)$$

and

$$\left(\frac{l}{r}\right)^2 = \frac{4}{c^2}\left(\frac{K}{3}\right)^2$$

In the same way we obtain from equation (1):

$$\left(\frac{l}{r}\right)^2 = \frac{E\,a}{\frac{1}{3}K}$$

and also

$$\frac{l}{r} = \sqrt{\frac{3\,E\,a}{K}} \tag{3b}$$

From either of these equations (3) or (3*b*) the length-ratio $\frac{l}{r}$ at the point of tangency can be ascertained. For greater ratios equation (1) must be used, and for smaller ratios equation (2).

From equations (3) and (3*b*), noticing that $\frac{l}{r}$ is common to both, we obtain

$$\frac{E\,a}{\frac{K}{3}} = \frac{4}{c^2}\left(\frac{K}{3}\right)^2$$

from which

$$a = \frac{4}{c^2\,E}\left(\frac{K}{3}\right)^3 \tag{4}$$

and

$$c = \sqrt{\frac{4}{a\,E}\left(\frac{K}{3}\right)^3} \tag{5}$$

With these two equations before us, we may either determine a empirically from the experimental lines and calculate the corresponding value of c; or we may reverse the process, determining c empirically and calculating a. By the latter process, which was first tried, the following values of c were obtained from the experimental lines on the diagrams:

$$c = \frac{2}{3}\sqrt{\frac{l}{E}\left(\frac{K}{3}\right)^3} \text{ for round ends} \tag{6}$$

$$c = \frac{1}{2}\sqrt{\frac{l}{E}\left(\frac{K}{3}\right)^3} \text{ for hinged ends} \tag{7}$$

$$c = \frac{2}{5}\sqrt{\frac{l}{E}\left(\frac{K}{3}\right)^3} \text{ for flat ends} \tag{8}$$

and by substituting these values of c in equation (4) the factors E and K disappear, and we obtain

For round ends,		$a = 9$	(9)
" hinged	"	$a = 16$	(10)
" flat	"	$a = 25$	(11)

from which it appears that the value of a is a function of the end-bearing alone, and is entirely independent of the materials.

Navier and Weisbach both make Euler's formula.

$$P = \frac{E I \pi^2}{l^2}$$

from which $a = \pi^2 = 9.87$.

Accepting this as the correct figure for round-end columns, and determining from the experiments the values of a for other end-bearings in terms of π^2, we have

$$\text{For round ends,} \quad a = \pi^2 \quad = 9.87 \tag{12}$$

$$\text{" hinged "} \quad a = \tfrac{5}{3}\pi^2 = 16.45 \tag{13}$$

$$\text{" flat "} \quad a = \tfrac{5}{2}\pi^2 = 24.67 \tag{14}$$

and from these values of a equation (5) becomes:

$$\text{For round ends } c = 0.6366 \sqrt{\frac{l}{E}\left(\frac{K}{3}\right)^3} \tag{15}$$

$$\text{" hinged " } c = 0.4932 \sqrt{\frac{l}{E}\left(\frac{K}{3}\right)^3} \tag{16}$$

$$\text{" flat " } c = 0.4000 \sqrt{\frac{l}{E}\left(\frac{K}{3}\right)^3} \tag{17}$$

Two sets of values for a and c are therefore presented; the latter having the advantage of theoretical accuracy, at least as to round ends, while the former have the advantage of greater simplicity. The experiments agree equally well with either. For the purpose of this article the latter figures will be used, but for ordinary office requirements the simpler form is preferable.

I would also call attention to the sequence of the values of a first found; viz., 9, 16, 25. These may be written 3^2, 4^2, 5^2. Does not this adherence to the squares of the natural numbers indicate the existence of some fundamental law underlying the relations of the several forms of end resistance.

No value for fixed-end columns is given, for the reason that no experiments upon this class of end-bearings have been made. Mr. Christie, indeed, has given us a group of experiments which he has called fixed ends. In plotting they show but little departure from square ends. They all plot within the limits of the field for that class, generally above the average, but some below. Upon closer examination of the details of the arrangement for fixing the column, it appears that it consists of an enlarged square-end bearing clamped to the test piece. These experiments, therefore, have not been treated

in this paper as a separate group, but have been classed as square ends, and so plotted.

These formulas can be readily applied to any material of which E and K are known. E is the modulus of elasticity, and needs no further comment. But what is K? Comparing the diagrams, Plates Nos. XLI, XLIV and XLV, with the various characteristics of iron, mild steel and hard steel, as given by Mr. Christie on page 262 of the Transactions, August, 1884, it would appear that K is not the resistance to direct compression, but is almost identical with the transverse modulus of rupture, as determined by Mr. Christie. But, on the other hand, Plates Nos. XLVI and XLVII, for cast-iron, show that it is not that modulus, but is more nearly allied to the modulus of compression, although by no means so great as the compressive resistance shown by short prisms.

Considering that the strain producing failure is one of compression, it would seem that the modulus of compression should be the one to govern the case.

In the tabular statements of Mr. Christie's experiments, there occur a number of notes, which, taken in connection with a certain feature of the plotted results, may throw some light on the subject. Throughout those tabular statements one is struck by the frequency with which occurs the note "no failure at 50 000 pounds," and that too, even with length ratios up to and greater than 200 radii. This result is analogous to that in which water, perfectly quiescent and free from any influence to disturb the molecular equilibrium, may be reduced very much below the freezing point without the formation of ice. In those experiments to which that note is attached, the test piece must not only have been perfectly fitted at the ends and accurately centered, but the test piece must have been perfectly straight, and the material so thoroughly homogeneous, that the forces and resistances acting at each cross-section were in perfect equilibrium, so much so that an external disturbing force would be necessary to destroy that equilibrium before failure could begin.

Mr. Christie has simply been fortunate enough to obtain practical illustrations of the mathematical principle which meets one in the very beginning of an attempt to deduce a formula for the strength of columns by analytical methods. A bending moment is always the product of the applied force by its effective lever-arm. Given a straight column, with a force acting in the direction of its axis, the lever-arm is

0, hence the bending moment is 0, and conversely the sustaining power of the column (as regards flexure) is infinity. Before a bending moment can be determined mathematically, there must be a known deflection, whose ordinate is the lever arm through which the applied force acts. If the direction of the force is axial, and there be perfect equilibrium at every cross-section, there can be no deflection from which to create a bending moment. This mathematical condition was realized physically by Mr. Christie.

That "no failure" should occur at rare intervals, should not be a matter of surprise. Its frequency in Mr. Christie's work is proof both of extreme care on his part, and unusual homogeneity of his material.

But if it is possible that this condition should occur in test pieces of relatively great length, we should expect it to occur with greater frequency in experiments on the shorter lengths. Now, turning again to Plate No. XLI, we note that within the length ratios from 0 to 30 radii of gyration, the experiments whose results lie above the upper limit of the field are quite numerous. If the lower limit showed a corresponding upward course, it would indicate a variation in the law as applied to the shorter lengths. But as the lower limit maintains a continued straight course, we assume that those above the upper limit are exceptional, and they are, without doubt, due to the same causes that enabled Mr. Christie to write "no failure" so frequently.

The question naturally arises: How far has this cause operated in all experiments upon the crushing strength of materials in short prisms, and given us false values for their moduli? Is it not altogether probable that the values of K, as indicated by these diagrams, are the true moduli of the compression resistances of these materials; and that the moduli of other materials as now published and accepted are correspondingly too high?

The principle announced by Hodgkinson, and reaffirmed and extended by Mr. C. L. Strobel, M. Am. Soc. C. E., on page 104, Vol. XI, of these Transactions, that "the ultimate strength of columns under 26 to 40 diameters long, is independent of their length," is evidently not sustained when all the experiments are combined in one group; and these gentlemen were, no doubt, misled by the prevalence in the experiments before them, of a preponderance of results of the exceptional character here noted.

At and near the length ratio of 100 in plates Nos. XLI and XLII

there is also a small group of experiments whose results reach above the upper limit. These most probably indicate a tendency in individual cases to persistently adhere to Euler's law, and so follow the prolongation of the curve beyond the point of tangency. It will be seen in the formula, that of the properties of the material, the elasticity alone determines the behavior of the column in the curved portion of the field, and in the tangential portion that behavior is determined by the combined influence of the moduli of elasticity and compression, the relative influence of K being greatest when $\frac{l}{r} = 0$ and diminishing to 0 at the point of tangency. It may therefore readily be supposed that near the point of tangency where the influence of K is least, it may be modified, or wholly suppressed, by accidental causes; such, for instance, as internal strain in the material, thus allowing the final result to be determined by the law of the curve instead of the law of the tangent.

The foregoing formulas and their constants were deduced from a comparison of the lines representing the average results of the several groups of experiments. If the law govering the strength of columns is correctly represented by the formulas, then by assigning to E and K their maximum and minimum values, the formulas should define the maximum and minimum limits of the field in the diagrams.

The values of E for wrought-iron are: Maximum, 38 000 000; minimum, 16 000 000; average, 27 000 000.

The values of K, as ascertain from Plates Nos. XLI, XLII and XLIII, are : Maximum, 50 000; minimum, 34 000; average, 42 000.

These maximum and minimum values applied to the formulas, give the limiting lines which have been drawn on Plates Nos. XLI, XLII and XLIII. It will be seen that a few of the experimental results lie beyond these limits. Those above the upper limit have been already accounted for, except some 6 or 8 on Plate No. XLII, hinged ends. These may have been due to some unrecorded condition of the pin-bearings giving rise to increased friction, and so making an end resistance more nearly allied to that of square ends. Those below the lower limit do not show local grouping to indicate special causes, as do the higher ones. But they can be explained by a cause which would act equally throughout all the length ratios, and one entirely consistent with the formulas and with observed results during tests. If the specimen is not properly placed

in the testing machine, the line of pressure will lie outside of the axis, and the distance between the axis of the column and the line of pressure becomes a lever arm by which the applied force is multiplied in its effect upon the column. In such cases, undetected, the recorded result would be unduly low, not because the column was weak, but because the effective force was greater than the recorded applied force. The very limited number of defective results of this class is better fitted to create confidence in the experimenters than to discredit deductions made from their work.

Having obtained the foregoing formulas from the published experiments on wrought-iron and steel, let us see how they will conform to the experiments on other materials. In Plates XLVI and XLVII, Mr. Hodgkinson's experiments on cast-iron columns are given, for square ends and round ends respectively; and also the lines of the formulas, giving to E for cast-iron its value of 16 000 000. The value of K for cast-iron, indicated by those experiments, is 80 000 (75 per cent. of the modulus of compression), and the resulting lines agree with the experiments.

Plate No. XLVIII shows some experiments on seasoned oak made by Lamande. They are taken from "Appleton's Dictionary of Engineering," article "Materials." Although the number and range of these experiments is quite limited, their harmony with the formula is very satisfactory. Here $K = 5\,400$, which is also about 75 per cent. of the modulus of compression.

It is a matter of regret that the extended series of experiments made some years since by Mr. C. Shaler Smith, M. Am. Soc. C. E., upon white and yellow pine have never been published. They would have afforded a valuable check upon these formulas, besides contributing largely to our general knowledge of these materials which are in daily use.

Perhaps the most extreme test of the formulas would be their application to building stones, whose physical properties present a greater contrast to those of iron and steel than is afforded by any other building material. The writer hopes at no distant day to lay before this Society a series of tests upon long prisms of oolitic limestone of Indiana, this stone being selected for the reason that its moduli have been fully determined, and are as follows:

Modulus of compression	12 600
" elasticity	4 350 000

Modulus of rupture (transverse)..............	2 340
Value of K (probably 75 per cent. of 12 000)...	9 000

(See Indiana Geol. Rep., 1881.)

In this paper no attempt has been made to trace the influence of the details of construction, relative size of pins to columns, limiting thickness of thinnest plates, etc., the aim having been to investigate only the general law. Judging from the fact that nearly all the experiments heretofore made are included within the limits due to the extreme variations in the quality of the material, it would seem that the other conditions enumerated are of secondary importance, and belong to that class of refinements of theory which cannot be utilized in practice, because they are dominated and obscured by the variations in the material. This statement must be taken, however, with the proviso that in all riveted work the pitch must be so proportioned to the thickness of plates that the strength per square inch of the plates between rivets shall not be less than the strength of the column considered as a whole.

Before closing this article, it will be well to revert to the older formulas and the reasons why they fail to give satisfactory results. It is apparent from the Plates, especially Nos. XLI and XLII, that Hodgkinson's experiments were too few in number to lead to correct results. Gordon fell into error, not only from the paucity of experiments at his command, but also by following Tredgold's assumption that a certain portion of the area is required to resist flexure. Under this assumption the modulus of compression would remain as a factor in the equation for all length ratios, but with diminishing influence, becoming zero only when $\frac{l}{r}$ becomes infinite; whereas it now appears that the modulus of compression ceases to be a factor when $P = \frac{K}{3}$. Hatzel following in the footsteps of Tredgold and Gordon was also led to equally erroneous results.

With the present formulas the construction of a diagram is very simple. A sufficient number of points having been calculated to plot the curves, a tangent line can be readily drawn to the point representing the value of K. All the experiments on steel indicate that a common value of E (27 000 000) will apply for wrought-iron and for all grades of steel. Hence, having constructed a diagram for wrought-iron, any grade of steel may be represented on it by drawing the tangent line to the point representing the proper value of K. A series of experiments to determine

the relation existing between the value of K and the percentage of carbon might lead to interesting and useful results.

Plate No. XLIX illustrates the relations between wrought-iron, mild steel and hard steel (carbon = 0.12 and 0.36 respectively) when used as flat-ended columns; the metal being assumed to have a uniform value of $E = 27\,000\,000$ pounds for all grades. On page 264 of the Transactions for 1884, Mr. Christie points out, as one result of his experiments, the practical equality of the resistance of these three metals for the higher length ratios. This diagram shows that the formulas herein presented fully recognize that fact.

Inasmuch as all experiments upon both large and small sections are here grouped together, and show no differences except those due to ordinary variations in the material, it is evident that a common law governs all the sizes tested, and we may return to the smaller and less costly test pieces with ample assurance that if only they are sufficient in number, the result will be equally valuable with those obtained from full-sized members.

The foregoing general formulas, with special values for different materials and end-bearings reduced to the simplest form, are appended in tabular form. The formulas for mild and hard steel, as given in the table, will be found to differ from the corresponding diagrams in the value assumed for E. The object of the diagrams being to aid in establishing the formulas, E was taken at 30 000 000 as most nearly representing the quality of the metal used in the experiments. (See Mr. Christie's papers). On the other hand the formulas in the tables are offered for general use, and E was taken at 27 000 000, which will most probably be found to be the general average for all qualities, both of wrought-iron and steel.

Column No. 6 of the table gives the length ratio for the point of tangency; or, in other words, the limiting length for the application of the first formula. All, or nearly all, of the dimensions occurring in ordinary practice will be within these limits, and the labor of calculation will be much simplified by the new formulas.

Through the kindness of Mr. Joseph M. Wilson, M. Am. Soc. C. E., I am permitted to use two of the plates accompanying his paper on "Bridge Specifications," on which he had plotted the lines representing the different formulas that have been proposed from time to time. On these plates, Nos. L and LI, I have added the lines of the formulas

Special Forms of the Equations for Various Materials and End-Bearings.

Materials.	E.	K.	End Bearings.	For Tangent. $\frac{P}{A} = K - c\frac{l}{r}$	At P. T. $\frac{l}{r} = \sqrt{\frac{3Ea}{K}}$	For Curve. $\frac{P}{A} = \frac{Ea}{\left(\frac{l}{r}\right)^2}$
Wrought-iron.	27 000 000	42 000	Flat.	$\frac{P}{A} = 42\,000 - 128\frac{l}{r}$	218.1	$\frac{P}{A} = \frac{666\,090\,000}{\left(\frac{l}{r}\right)^2}$
			Hinged.	$\frac{P}{A} = 42\,000 - 157\frac{l}{r}$	178.1	$\frac{P}{A} = \frac{444\,150\,000}{\left(\frac{l}{r}\right)^2}$
			Round.	$\frac{P}{A} = 42\,000 - 203\frac{l}{r}$	138.0	$\frac{P}{A} = \frac{266\,490\,000}{\left(\frac{l}{r}\right)^2}$
Mild steel.... (Carbon = 0.12)	27 000 000	52 500	Flat.	$\frac{P}{A} = 52\,500 - 179\frac{l}{r}$	195.1	$\frac{P}{A} = \frac{666\,090\,000}{\left(\frac{l}{r}\right)^2}$
			Hinged.	$\frac{P}{A} = 52\,500 - 220\frac{l}{r}$	159.3	$\frac{P}{A} = \frac{444\,150\,000}{\left(\frac{l}{r}\right)^2}$
			Round.	$\frac{P}{A} = 52\,500 - 284\frac{l}{r}$	123.3	$\frac{P}{A} = \frac{266\,490\,000}{\left(\frac{l}{r}\right)^2}$
Hard steel..... (Carbon = 0.36)	27 000 000	80 000	Flat.	$\frac{P}{A} = 80\,000 - 337\frac{l}{r}$	158.0	$\frac{P}{A} = \frac{666\,090\,000}{\left(\frac{l}{r}\right)^2}$
			Hinged.	$\frac{P}{A} = 80\,000 - 414\frac{l}{r}$	129.0	$\frac{P}{A} = \frac{444\,150\,000}{\left(\frac{l}{r}\right)^2}$
			Round.	$\frac{P}{A} = 80\,000 - 534\frac{l}{r}$	99.9	$\frac{P}{A} = \frac{266\,490\,000}{\left(\frac{l}{r}\right)^2}$
Cast-iron......	16 000 000	80 000	Flat.	$\frac{P}{A} = 80\,000 - 438\frac{l}{r}$	121.6	$\frac{P}{A} = \frac{394\,720\,000}{\left(\frac{l}{r}\right)^2}$
			Hinged.	$\frac{P}{A} = 80\,000 - 537\frac{l}{r}$	99.3	$\frac{P}{A} = \frac{263\,200\,000}{\left(\frac{l}{r}\right)^2}$
			Round.	$\frac{P}{A} = 80\,000 - 693\frac{l}{r}$	77.0	$\frac{P}{A} = \frac{157\,920\,000}{\left(\frac{l}{r}\right)^2}$
Oak............	1 200 000	5 400	Flat.	$\frac{P}{A} = 5\,400 - 28\frac{l}{r}$	128.1	$\frac{P}{A} = \frac{29\,604\,000}{\left(\frac{l}{r}\right)^2}$
Oolitic Limestone ..	4 350 000	9 000	Flat.	$\frac{P}{A} = 9\,000 - 32\frac{l}{r}$	189.1	$\frac{P}{A} = \frac{107\,314\,500}{\left(\frac{l}{r}\right)^2}$

herein presented, which are thus shown in comparison with the others heretofore proposed. The relations of the several lines are so clearly shown on the plates, that no special discussion of them seems to be required.

DISCUSSION.

JAMES CHRISTIE, M. Am. Soc. C. E.—If the compressive force to which columns are subjected acted through axes whose position could be accurately predetermined, we would probably obtain more concordant results from experiment than we do, and no doubt could frame formulas, based on known physical properties of materials, which could be depended on as reliable for any length or section of column.

But, in addition to the fact that the center of stress may be acting through an axis which no care in design or manufacture can accurately foresee, we also have disturbing conditions, such as columns not perfectly straight; or, if apparently straight externally, the material at different cross-sections may be so distributed that the centers of symmetry may not coincide on any straight line.

Again, by reason of internal molecular stresses, or other unknown causes, it is questionable if the modulus of elasticity, especially for metals in compression, is uniform throughout the mass. When investigating the theory of column resistance we assume that the curve of flexure takes some regular deflection in a certain direction, whereas experiment shows that frequently several points of contra-flexure may occur in a single column, due to the sinuosity of the centers of maximum resistance. It is quite possible that this condition exists in such a large number of cases as to render it a prominent circumstance in determining the average resistance by experiment. Therefore it would seem probable that a general scale of averages derived from experiment would be a safer guide than any formula based on fundamental principles, which may be derived from theoretical assumptions which rarely obtain in practice. Having obtained a satisfactory table of average extreme resistances, it becomes the duty of the prudent engineer to adopt such working factors as will safely include the weak, erratic cases which will occasionally occur, despite ordinary care, in design and construction.

The formulas proposed by Mr. Johnson possess the merit of being fitted to a line which expresses the average resistance of known experiments. I think, however, that the value assigned to K is too small, for an examination of the diagram for flat ends of wrought-iron shows a marked number of tests of the shortest columns which lie above the maximum limit.

Assuming K to represent the modulus of compression, it might be more proper to locate the vertical axis of the diagram at some definite length instead of at zero, for the compression resistance of infinitely short specimens of a ductile metal becomes practically infinite.

I cannot understand why we should assume that the influence of K should disappear at the specific point assigned. Would it not be more reasonable to assume that the line of resistance approached nearer to the elastic curve as the column became longer, and receded further from it or approached the tangent as the column became shorter? This, however, would mar the simplicity of Mr. Johnson's formulas, and, as practical accuracy rather than a mathematical refinement is the object sought, we may accept any satisfactory formula, which, after all, can only be a compromise between so many conflicting elements.

The strength of columns within the range of lengths usual in practice, is determined more directly by resistance to crushing than by lateral stiffness. These columns are covered by Mr. Johnson's equation for the tangent, which is exceedingly simple, and admits of ready application. This observation applies more especially to metals of low tenacity. For instance, the elastic limit for compression of wrought-iron (about 29 000 pounds) will usually be exceeded in pressure before failure occurs on flat-ended columns having a ratio of l to r of 100 and under. In the same manner a soft steel having an elastic limit of 38 000 pounds will cover a range of $\frac{l}{r}$ up to 80. Hard steel of 100 000 pounds tenacity, or 60 000 pounds elastic limit, will usually cover a range of $\frac{l}{r}$ up to 60 before failure ensues from 60 000 pounds pressure per square inch.

I append herewith the results of a few tests of riveted models of iron and steel I have made recently. These specimens were each formed of two angle sections of the dimensions denoted, riveted together, side to side, forming a T-shaped section.

The holes were punched and fastened with ⅝-inch rivets, 4 inches apart. Iron rivets being used on iron bars and steel rivets on steel. No care was taken to prevent the specimens being exposed to the roughest treatment usual in the operations of punching, straightening, etc. The mild steel was of 60 000 pounds tensile strength, and the hard steel 100 000 pounds.

Failure was normal in every case—that is, true flexure occurred without fracture. As the rivet holes were unduly large as compared with the sections operated upon, it is probable that the specimens were weakened more than was needful.

COMPRESSION TESTS OF FLAT-ENDED RIVETED MODELS.

Kind of Metal.	Size of Each Angle Composing the Section.	Length in Inches.	Sectional Area in Square Inches.	Maximum Resistance in Pounds.	Maximum Resistance in Pounds per Square Inch.	$\frac{l}{r}$
	Inches.					
Iron	$1\frac{1}{2} \times 1\frac{1}{2} \times \frac{5}{16}$	$47\frac{7}{8}$	1.74	44 100	25 350	107
"	$1\frac{1}{4} \times 1\frac{1}{4} \times \frac{5}{16}$	$36\frac{1}{16}$	1.39	34 325	24 700	97
"	$1\frac{1}{8} \times 1\frac{1}{8} \times \frac{5}{16}$	$24\frac{1}{32}$	1.22	35 525	28 890	75
"	$1\frac{3}{8} \times 1\frac{3}{8} \times \frac{5}{16}$	$47\frac{23}{32}$	1.53	36 450	23 820	120
Mild steel	$1\frac{3}{8} \times 1\frac{3}{8} \times \frac{5}{16}$	48	1.54	48 425	31 757	120
"	$1\frac{1}{4} \times 1\frac{1}{4} \times \frac{5}{16}$	$36\frac{3}{16}$	1.39	44 800	32 230	98
"	$1\frac{1}{8} \times 1\frac{1}{8} \times \frac{5}{16}$	24	1.22	46 100	37 790	75
"	$1\frac{3}{8} \times 1\frac{3}{8} \times \frac{5}{16}$	$42\frac{25}{32}$	1.54	46 525	30 220	102
Hard steel	$1\frac{1}{2} \times 1\frac{1}{2} \times \frac{17}{64}$	42	1.25	48 300	38 610	93
"	$1\frac{3}{16} \times 1\frac{3}{16} \times \frac{17}{64}$	$36\frac{1}{4}$	1.18	47 650	42 165	100
"	$1\frac{5}{16} \times 1\frac{5}{16} \times \frac{17}{64}$	$48\frac{1}{8}$	1.32	46 900	35 440	130
"	$1 \times 1 \times \frac{17}{64}$	24	.95	43 575	45 750	82

I think Mr. Johnson gives insufficient credit to the group of tests of fixed-ended struts when he classes them simply as flat-ended.

The flanges attached to the ends of the specimens were as effectual in aiding resistance to lateral flexure as though they were extended over any greater area.

The general phenomena observed at the period of failure was as follows:

For lengths up to about 30 times r, the specimens failed by irregular crippling.

From 30 to about 100 times r regular curvature occurred, but the flat ends remained seated after failure. Contrary flexure was manifest near

the ends, consequently no method of securing the ends could have materially increased the resistance.

Above the latter range of length the flat-ended specimens showed a tendency to rotate on their ends at the time of failure, and in the case of the longest bars tested, the specimens tilted on their ends before the maximum resistance to flexure was recorded, and the curve became uniform from end to end. With flanges attached (fixed-ended), the ends remained rigid, no rotation whatever ensued, and the reverse curves were as apparent on the longest specimens as on the shorter free-ended bars. As $\frac{l}{r}$ exceeded 150, the resisting influence of the flanges became very apparent, below that limit this influence gradually disappeared, becoming barely discernible with $\frac{l}{r}$ at 100.

THOS. H. JOHNSON, M. Am. Soc. C. E.—Mr. Christie has raised a question as to the value assigned to K in my formula, and I thank him for the opportunity thus afforded of saying some things omitted or imperfectly said in the original paper.

At $\frac{l}{r} = 0$ the resistance to compression is not infinity, but must theoretically be equal to the modulus of compression.

Practically of course, experiments at $l = 0$ are impossible, and when $\frac{l}{r}$ is very small, other resistances are brought into play, so that experimenters have long since recognized the importance of making the test pieces long enough to admit of the fractured portions moving freely without added resistance. To my mind the plotted experiments show very conclusively that other causes operate which are not eliminated by making the length of test piece for compression one or one and one-half times d, as is usually done; and that these causes are not wholly eliminated, and the normal modulus of compression obtained experimentally (on metals at least) until $\frac{l}{r} = 25$, which would be for rectangular sections $l = 7\frac{1}{4}d$ nearly.

In my paper I have explained the cause operating to produce these abnormal results, and at the same time account for the many instances in which Mr. Christie failed to destroy relatively very long columns. Whether or not the explanation there offered is satisfactory, the fact remains clearly shown by the experiments, that the modulus of com-

pression as derived from short prisms is too high to be used in a formula for columns.

In case of fibrous metals, in which the resistance to tension is equal to or greater than that to compression, transverse rupture occurs by failure on the upper or compression side of the beam; and for such materials the modulus of rupture should be coincident with the modulus of compression.

Comparing the moduli of rupture given by Mr. Christie on page 262 of Volume XIII of the Transactions of this Society with the values of K obtained from the diagrams, we have the following:

	Iron.	Mild Steel.	Hard Steel.
Modulus of rupture.....	44 800	52 900	80 200
Value of K.............	42 000	52 500	80 000

For materials which do not fail by compression at the upper side, the modulus of rupture is usually a compound function of the moduli of tension and compression, so that the results of transverse experiments cannot be compared with the values of K as used in the formulas.

Hodgkinson gives the modulus of compression for the cast-iron used by him in his experiments at 109 800 pounds. The diagram shows that the value of K which best conforms to his experiments is 80 000 pounds, which is 73 per cent. of the modulus obtained from short prisms.

The modulus of compression for oak is about 7 500 pounds, and the value of K from the diagram is 5 400 pounds, which is also 73 per cent. of the modulus obtained from short prisms. I have therefore assumed (provisionally) that in applying the formulas to materials in which experiments on long columns are wanting, it will be proper to assume the value of K at about 75 per cent. of the modulus of compression as obtained from short prisms.

Returning to the specific point made by Mr. Christie, that the value of K taken by me is too low, as evidenced by the diagram for flat ends, I would call attention to the fact that the value of K is a property of the material entirely independent of the end resistance, and must apply alike to all forms of end bearings. In fixing its value from the diagrams,

all of the forms of end bearings must be kept in view, and such value taken as will best fit all of them. This I endeavored to do. The figures given by me will bear modification, within certain limits, without greatly disagreeing with the experiments on the several forms of end bearings; but they appear to harmonize all the forms of end bearings to the best advantage.

In treating the fixed-end experiments as flat ends, I did not mean to assert that the flanged ends had no effect. They do show results somewhat higher than the average of the flat ends, but it seemed to me that the difference would not warrant a separate discussion of them; besides, the number of experiments was quite limited, especially in the curved part of the field.

The equation of the tangent for this group of experiments would be

$$\frac{P}{A} = 42\,000 - 123\,\frac{l}{r} \qquad \text{(a)}$$

and for the curve,

$$\frac{P}{A} = \frac{27\,Ea}{\left(\frac{l}{r}\right)^2} \qquad \text{(b)}$$

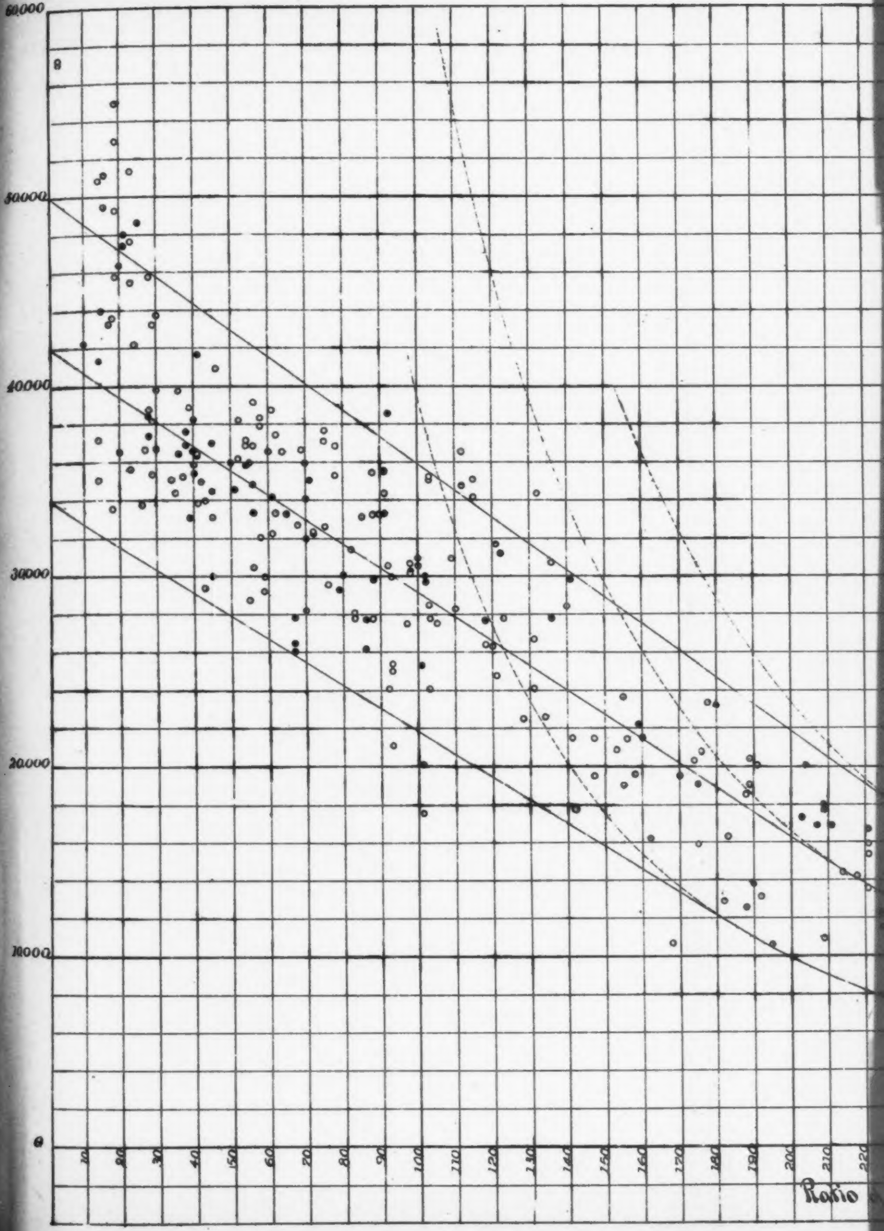
The point of tangency will occur at $\frac{l}{r} = 228$.

Eq. (b) will give results only about 8 per cent. higher than the equation for flat ends for length ratios greater than 228. For smaller ratios the percentage of gain diminishes in proportion to the length ratio.

Inasmuch as flanged columns are seldom used in engineering works, though more frequently in architectural structures, it would appear to be judicious to disregard the slight increase of strength due to the flange, and treat all such cases as square ends.

Euler's equation is a theoretical deduction made years ago, and the experiments prove that it is correct for the higher length ratios. The resistance to crushing is not a factor in that equation, and hence its influence must become zero at the point where that equation begins to apply. Why this should be so is difficult to see in the light of our present knowledge; but the evidence goes to show that it is a fact.

WROUGHT IRON



FLAT ENDS.

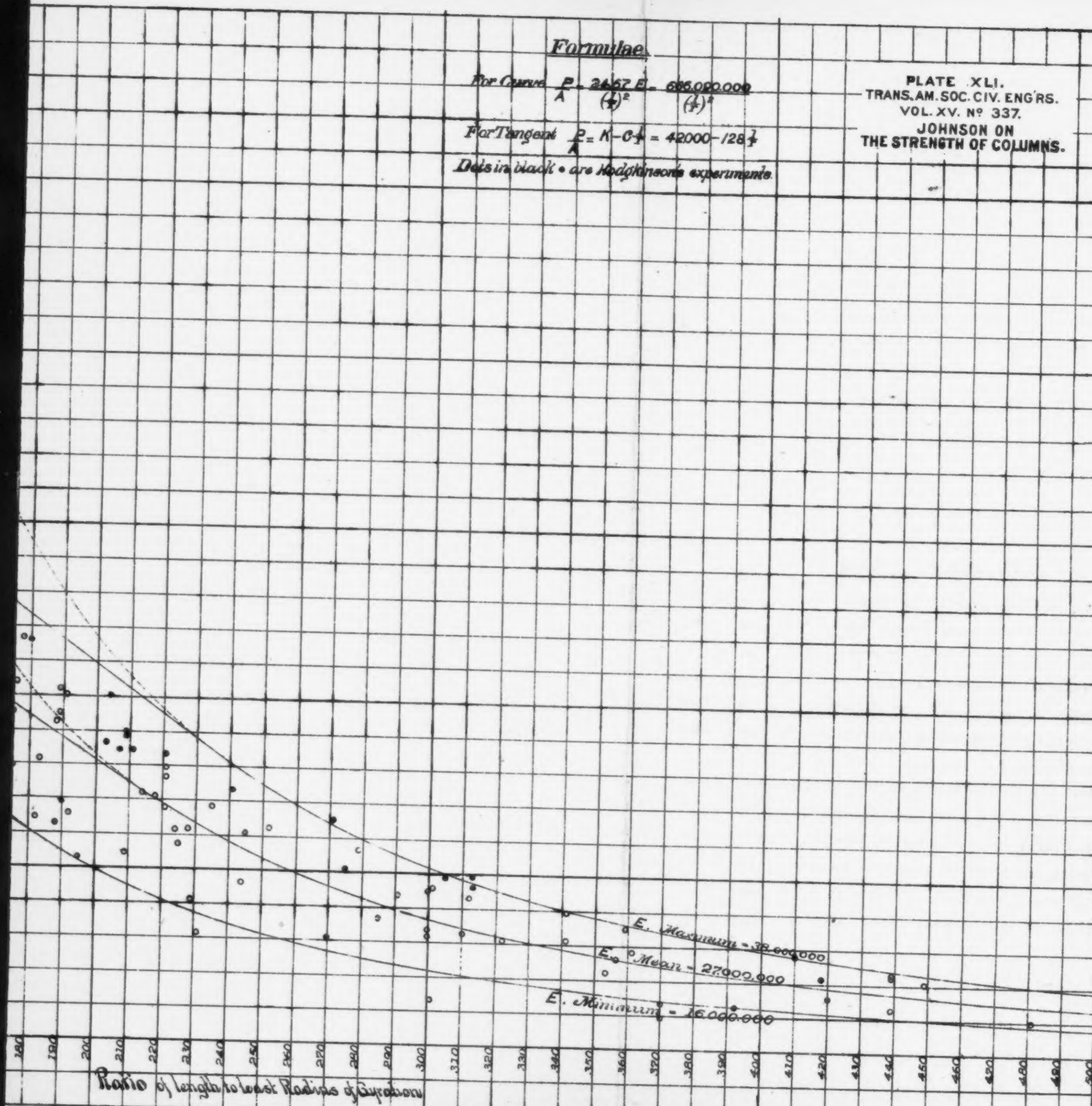
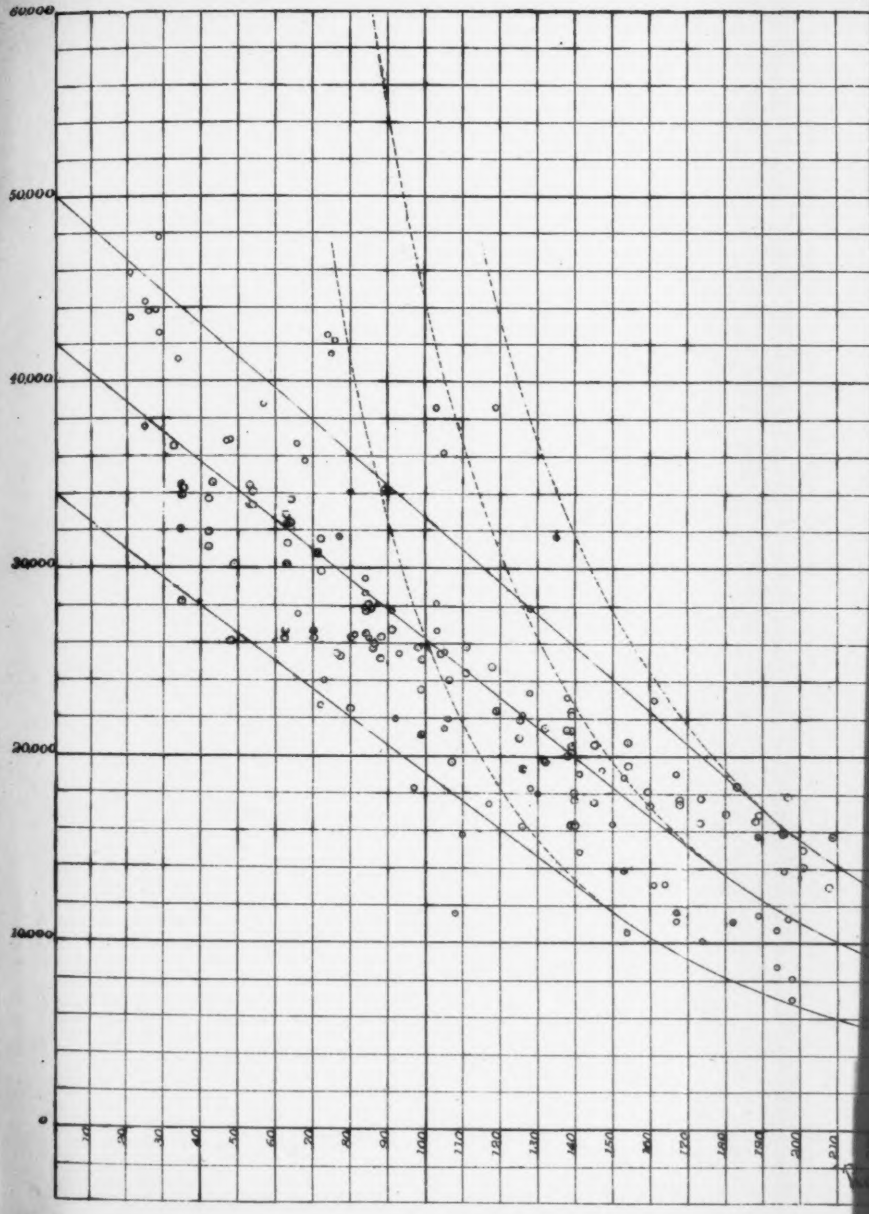


PLATE XLI.
TRANS. AM. SOC. CIV. ENG'RS.
VOL. XV. Nº 337.
JOHNSON ON
THE STRENGTH OF COLUMNS.

WROUGHT IRON.

HI



HINGED ENDS.

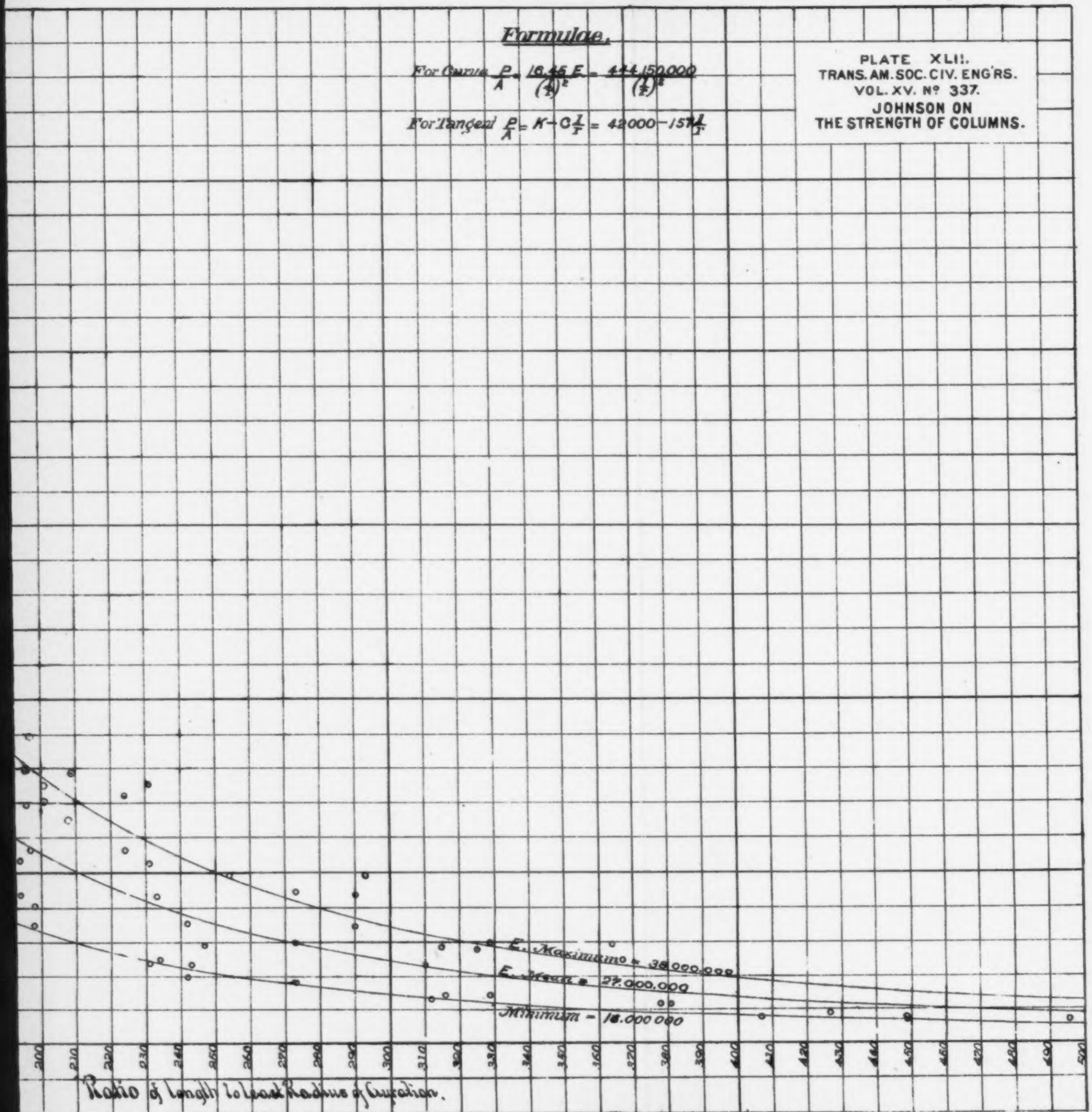
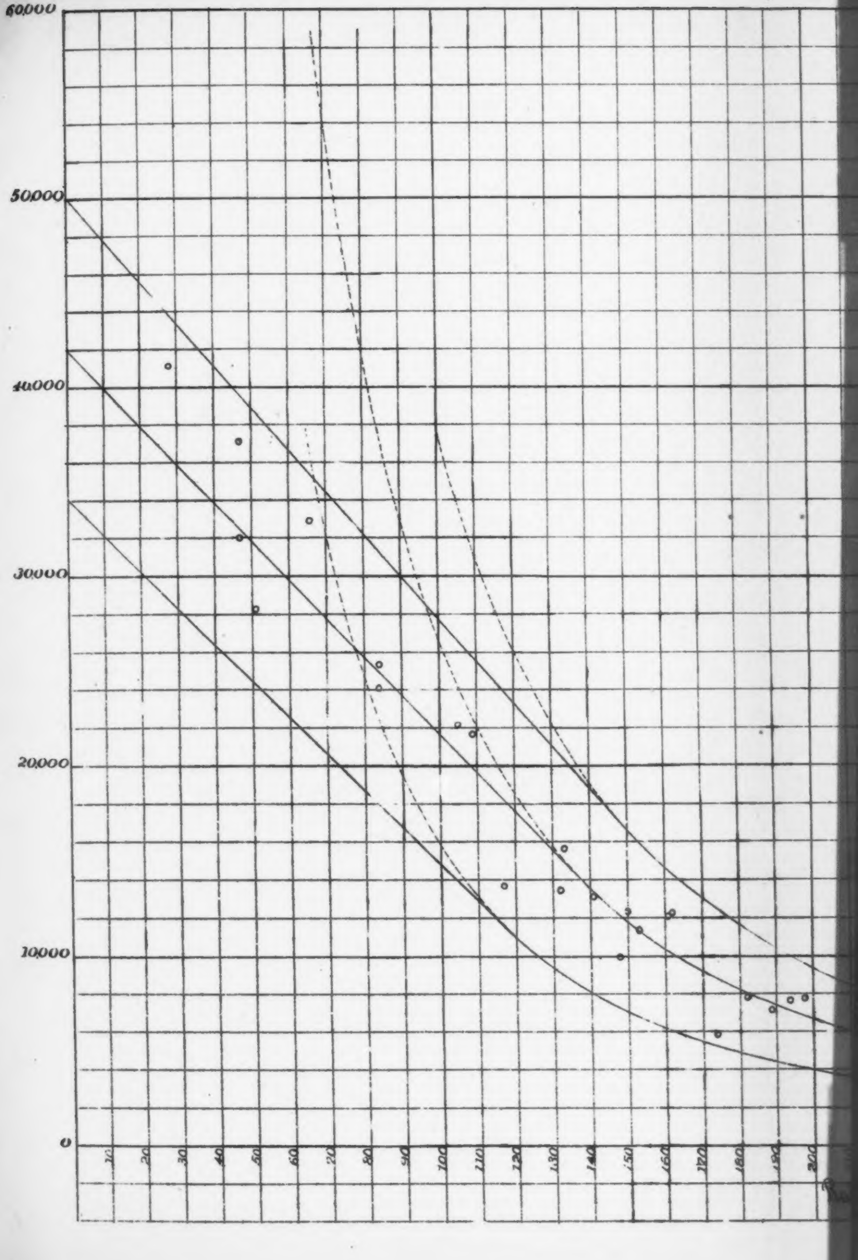
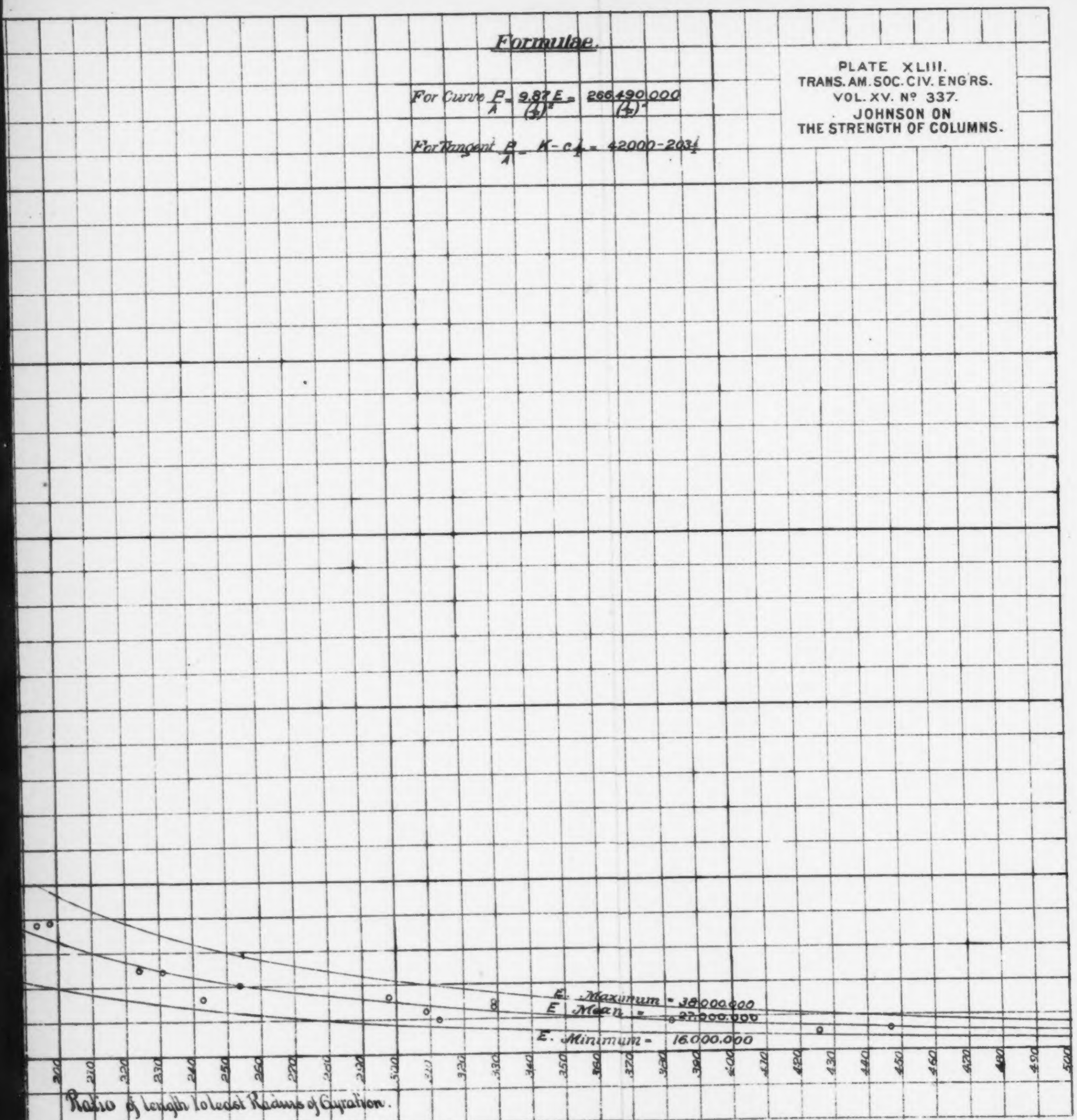


PLATE XLII.
TRANS. AM. SOC. CIV. ENG'RS.
VOL. XV. No. 337.
JOHNSON ON
THE STRENGTH OF COLUMNS.

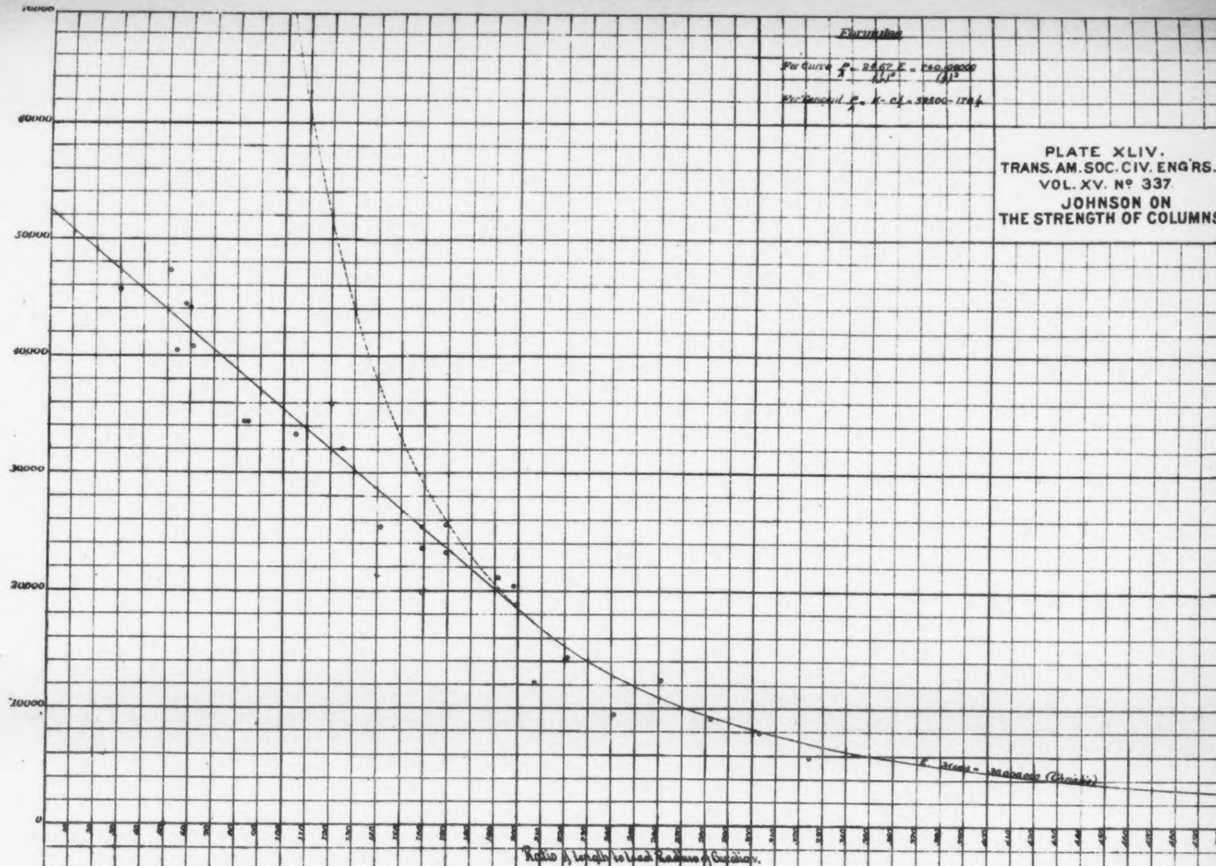
WROUGHT IRON.



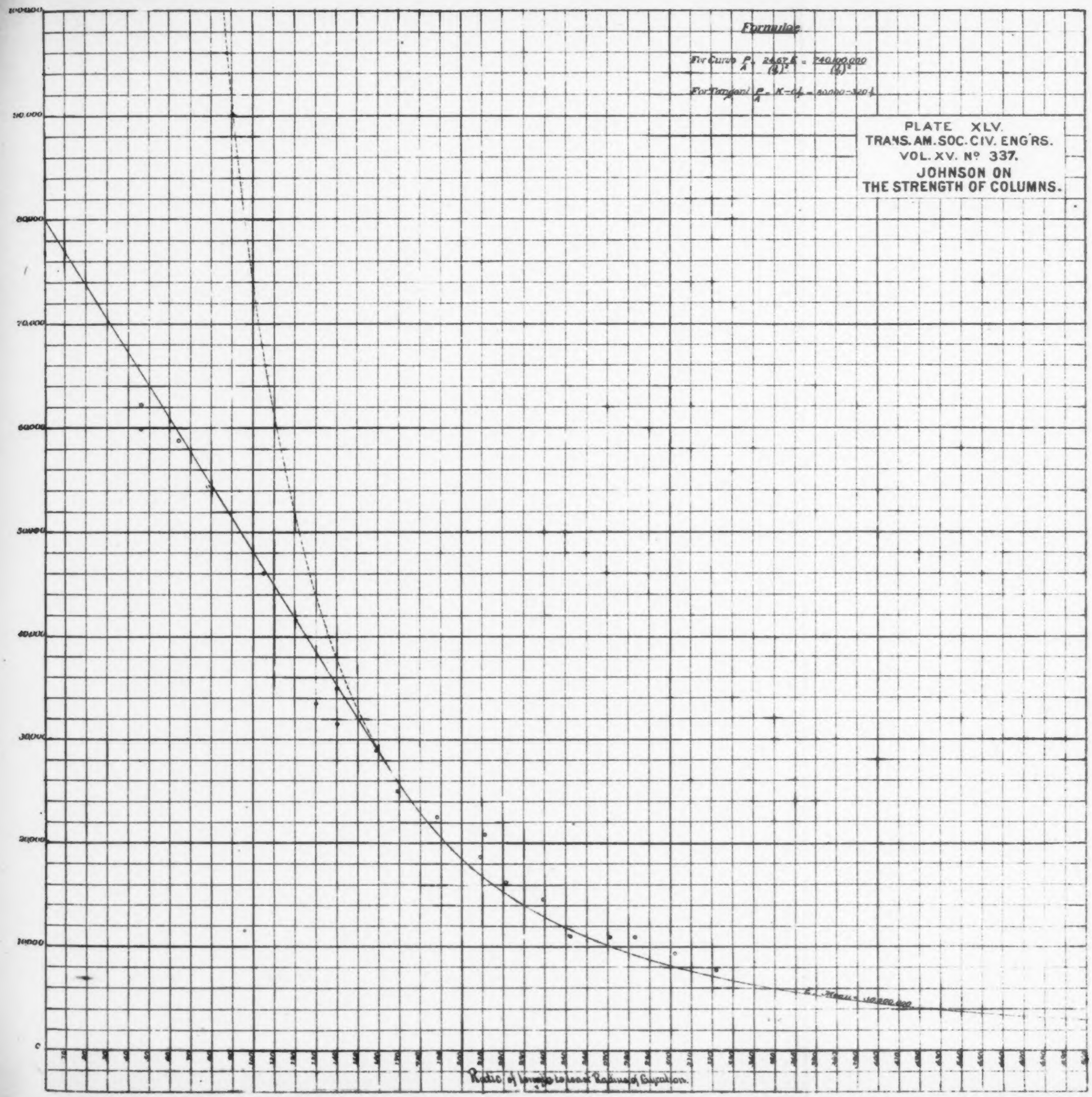
ROUND ENDS


Formulae.
For Curve P/A = 9.87 E/(l/r)² = 266490000/(l/r)²
For Tangent P/A = K - c l/r = 42000 - 203 l/r
PLATE XLIII.
TRANS. AM. SOC. CIV. ENGRS.
VOL. XV. № 337.
JOHNSON ON
THE STRENGTH OF COLUMNS.
E. Maximum = 38.000.000
E. Mean = 27.000.000
E. Minimum = 16.000.000
200
210
220
230
240
250
260
270
280
290
300
310
320
330
340
350
360
370
380
390
400
410
420
430
440
450
460
470
480
490
500
Ratio of length to least Radius of Gyration.

MILD STEEL. FLAT ENDS

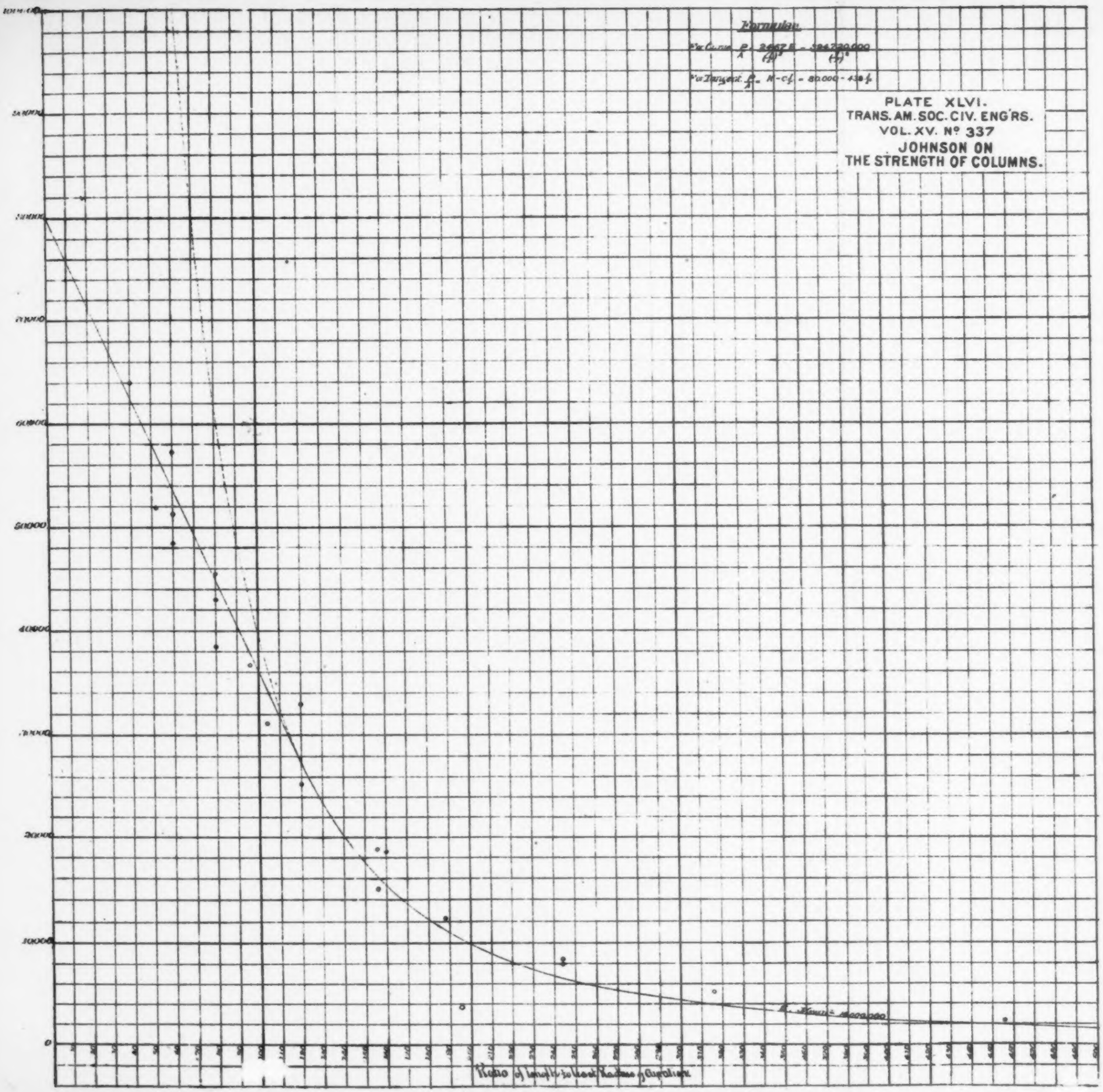


HARD STEEL **FLAT ENDS**



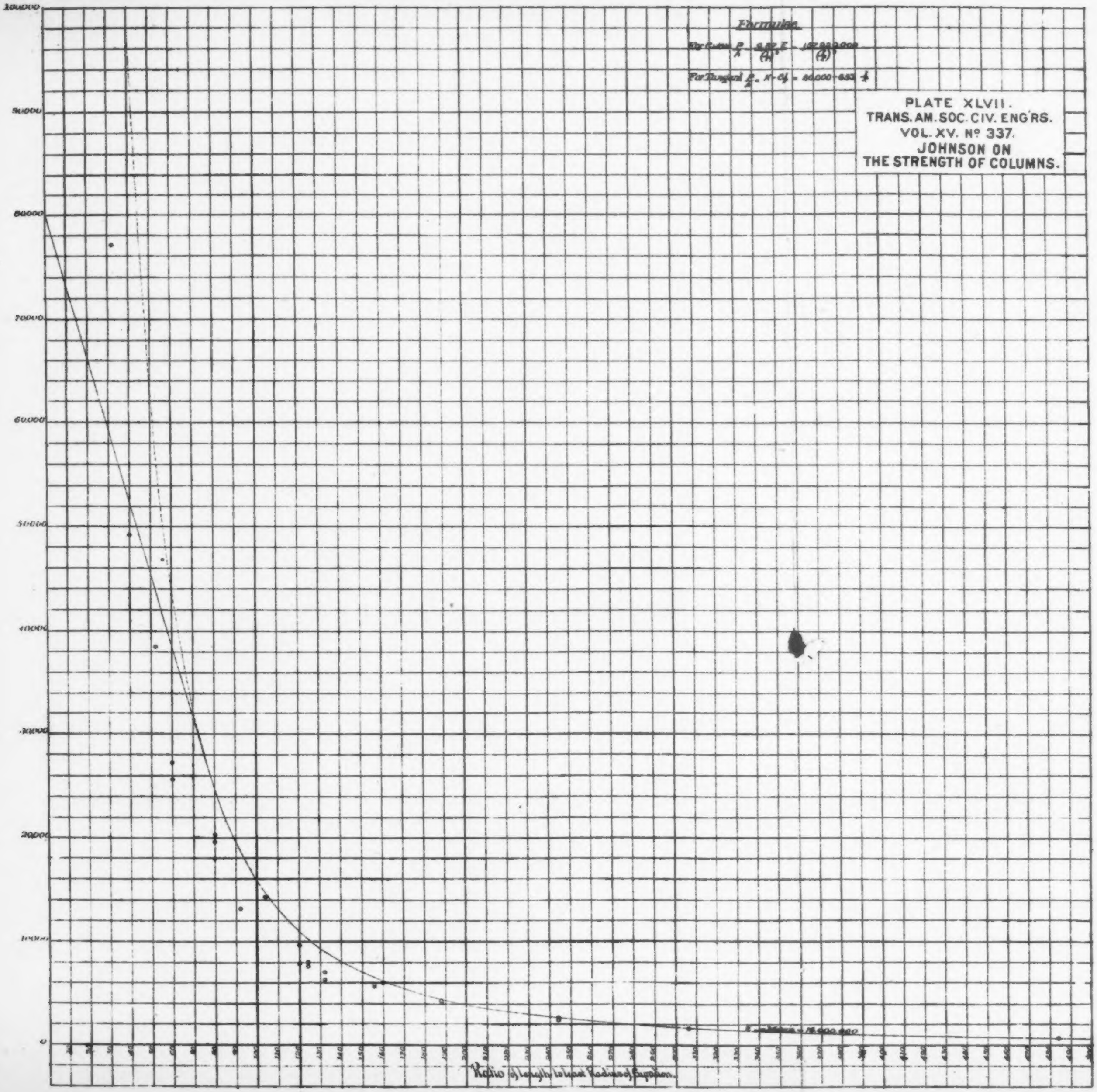
CAST IRON

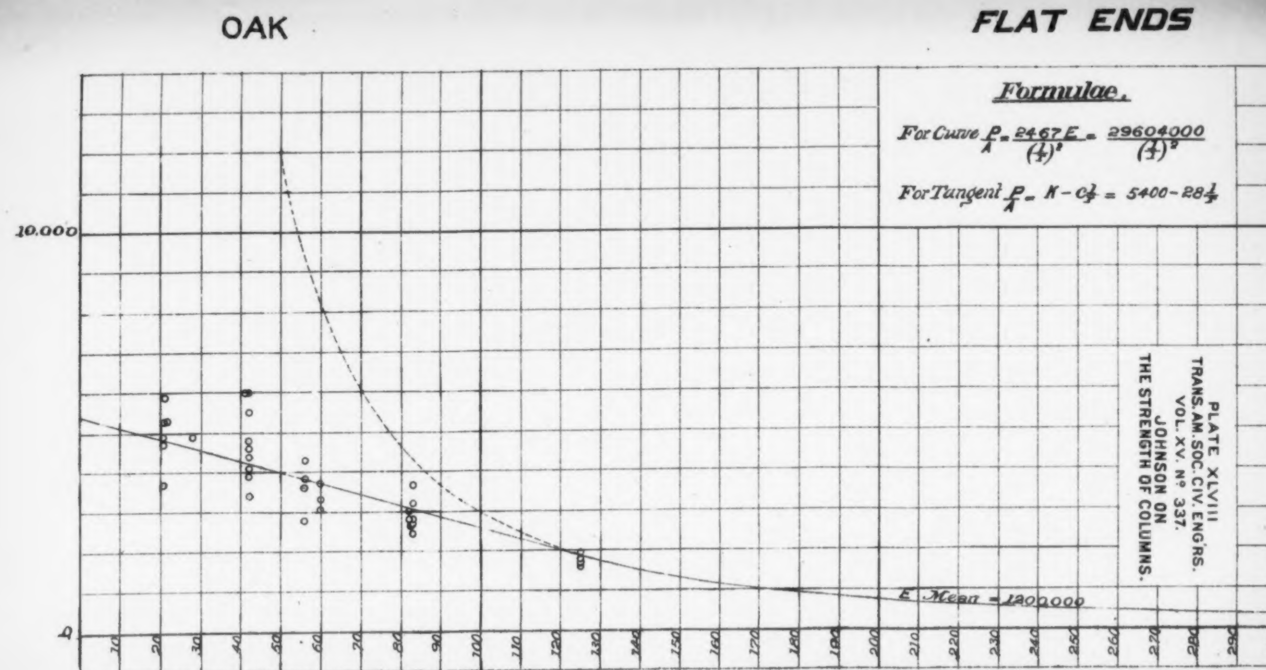
FLAT ENDS.



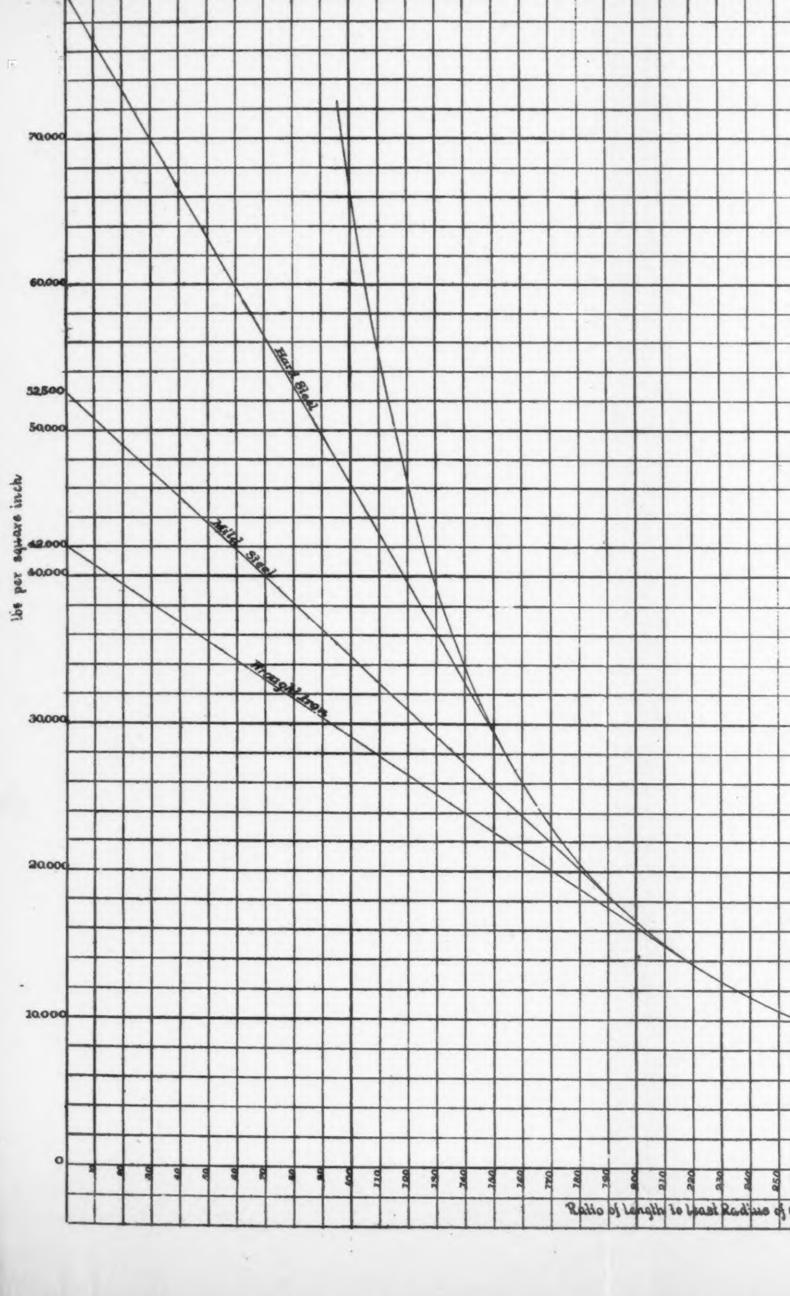
CAST IRON

ROUND ENDS.





OAK
FLAT ENDS
Formulae.
For Curve $\frac{P}{A} = \frac{2467E}{(\frac{l}{r})^2} = \frac{29604000}{(\frac{l}{r})^2}$
For Tangent $\frac{P}{A} = K - C\frac{l}{r} = 5400 - 28\frac{l}{r}$
PLATE XLVIII
TRANS. AM. SOC. CIV. ENG'RS.
VOL. XV. Nº 337.
JOHNSON ON
THE STRENGTH OF COLUMNS.
E Mean = 1200000
10.000
0
10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180 190 200 210 220 230 240 250 260 270 280 290 300


80,000
70,000
60,000
52,500
50,000
42,000
40,000
30,000
20,000
10,000
0
lbs per square inch
Hard Steel
Mild Steel
Wrought Iron
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
210
220
230
240
250
Ratio of length to least Radius of

FLAT ENDS

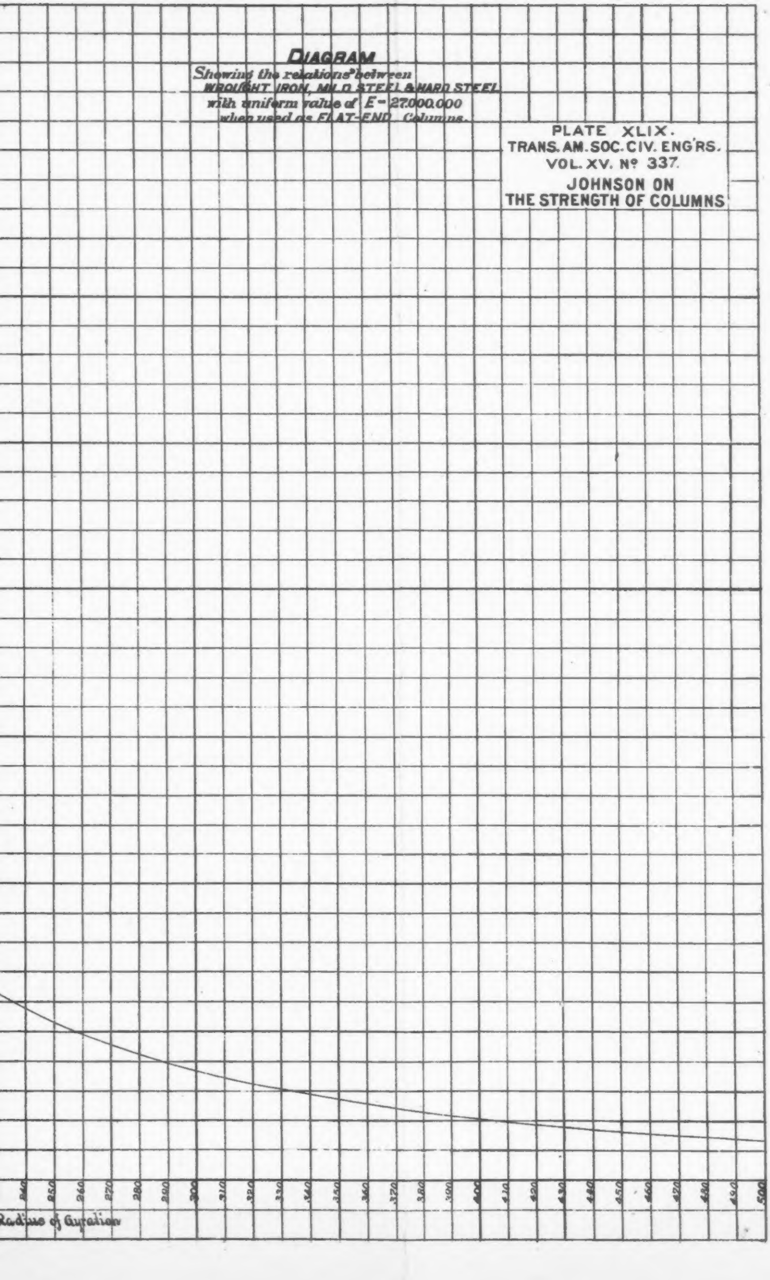
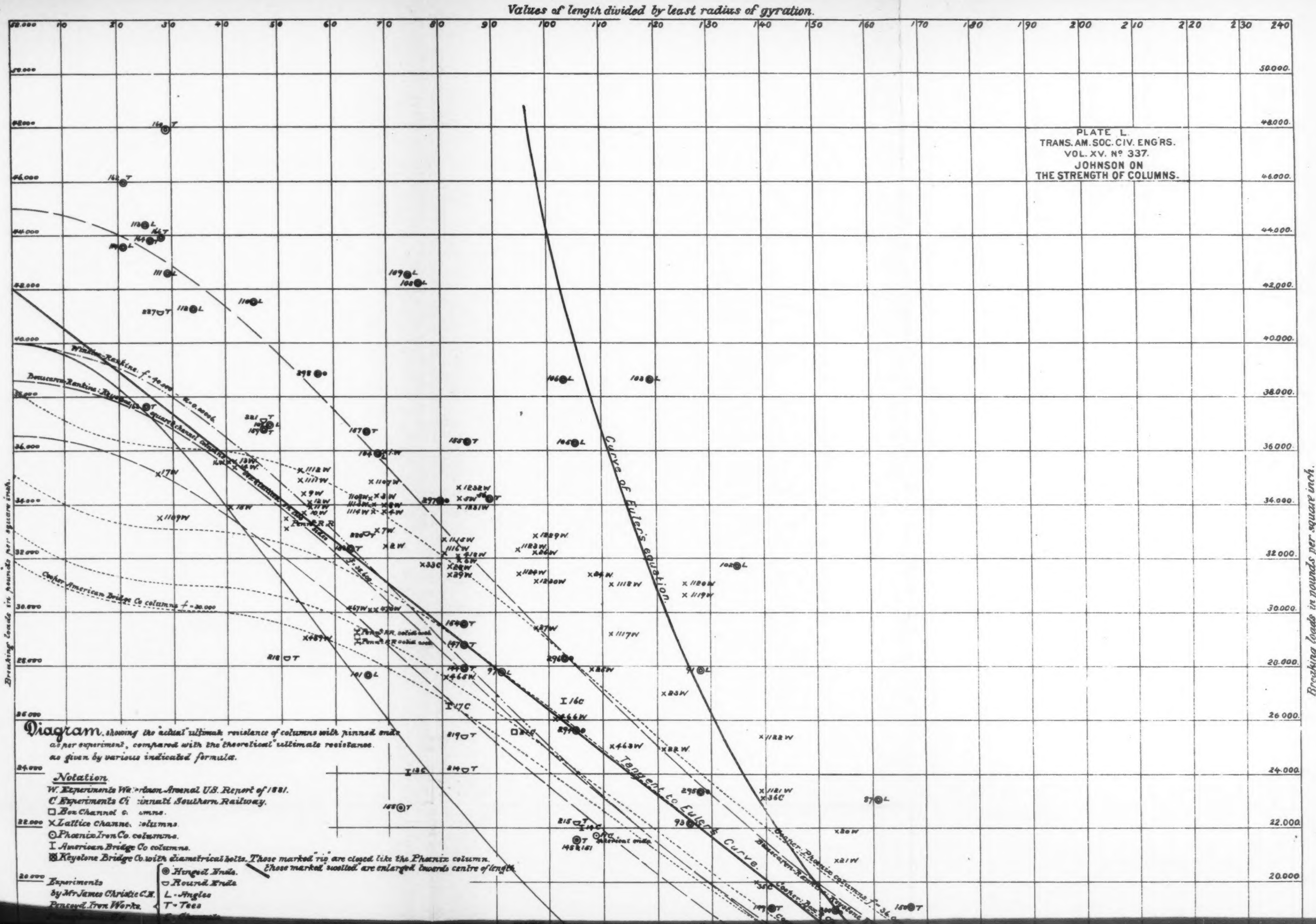
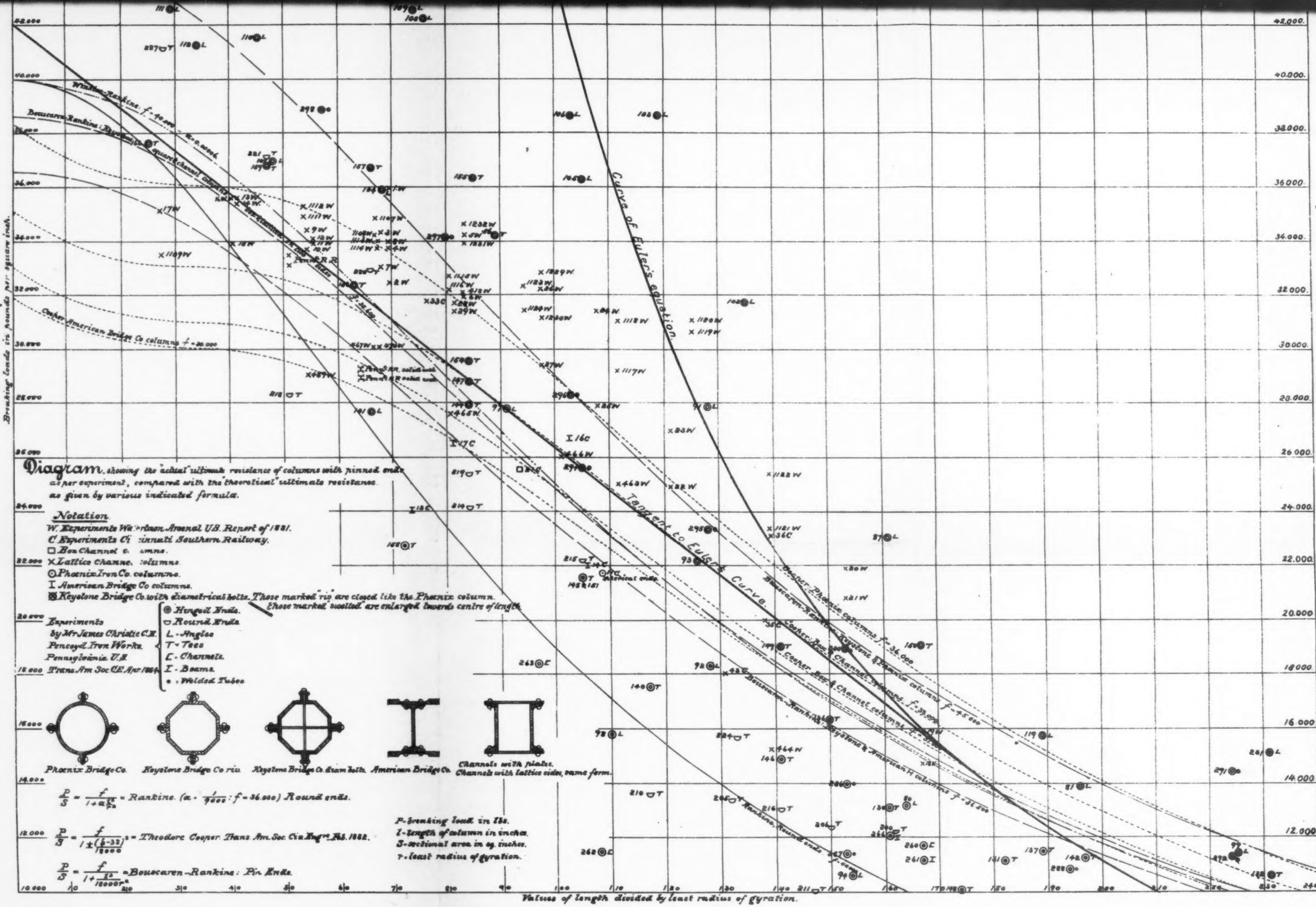


DIAGRAM
Showing the relations between
WROUGHT IRON, MILD STEEL & HARD STEEL
with uniform value of E = 27.000.000
when used as FLAT-END Columns.
PLATE XLIX.
TRANS. AM. SOC. CIV. ENG'RS.
VOL. XV. Nº 337.
JOHNSON ON
THE STRENGTH OF COLUMNS
240
250
260
270
280
290
300
310
320
330
340
350
360
370
380
390
400
410
420
430
440
450
460
470
480
490
500
Radius of Gyration



Values of length divided by least radius of gyration.
PLATE L.
TRANS. AM. SOC. CIV. ENG'RS.
VOL. XV. Nº 337.
JOHNSON ON
THE STRENGTH OF COLUMNS.
Breaking loads in pounds per square inch.
Curve of Euler's equation
Tangent to Euler's Curve.
Cooper American Bridge Co columns f = 30.000
Diagram, showing the "actual" ultimate resistance of columns with pinned ends as per experiment, compared with the "theoretical" ultimate resistance as given by various indicated formulæ.
Notation.
W. Experiments Wa rtown Arsenal U.S. Report of 1881.
C Experiments Ci innati Southern Railway.
Box Channel c umns.
Lattice Channe olumns.
Phœnix Iron Co. columns.
American Bridge Co columns.
Keystone Bridge Co. with diametrical bolts. Those marked "rio" are closed like the Phœnix column.
Those marked "swelled" are enlarged towards centre of length.
Hinged Ends.
Round Ends
L - Angles
T - Tees
Experiments
by Mr James Christie C.E.
Pencoyd Iron Works.



Diagram, showing the "actual" ultimate resistance of columns with pinned ends as per experiment, compared with the "theoretical" ultimate resistance as given by various indicated formulæ.
Notation.
W. Experiments Watertown Arsenal U.S. Report of 1881.
C. Experiments Cincinnati Southern Railway.
□ Box Channel columns.
× Lattice Channel columns.
⊙ Phoenix Iron Co. columns.
I American Bridge Co. columns.
⊠ Keystone Bridge Co. with diametrical bolts. Those marked "riv" are closed like the Phoenix column.
Those marked "swelled" are enlarged towards centre of length.
Experiments by Mr James Christie C.E. Pencoyd Iron Works, Pennsylvania U.S. Trans. Am. Soc. C.E. Apr. 1884.
⊙ Hinged Ends.
▽ Round Ends.
L - Angles
T - Tees
C - Channels.
I - Beams.
• - Welded Tubes
Phoenix Bridge Co.
Keystone Bridge Co. riv.
Keystone Bridge Co. diam. bolts.
American Bridge Co.
Channels with plates. Channels with lattice sides, same form.
$\frac{P}{S} = \frac{f}{1+a\frac{l^2}{r^2}}$ = Rankine. ($a = \frac{1}{9000}$: f = 36.000) Round ends.
$\frac{P}{S} = \frac{f}{1 \pm \left(\frac{\frac{l}{r}-33}{15000}\right)^2}$ = Theodore Cooper. Trans. Am. Soc. Civ. Engrs. Feb. 1882.
$\frac{P}{S} = \frac{f}{1+\frac{l^2}{18000 r^2}}$ = Bouscaren-Rankine: Pin Ends.
P - breaking load in lbs.
l - length of column in inches.
S - sectional area in sq. inches.
r - least radius of gyration.
Curve of Euler's equation.
Tangent to Euler's Curve
Winslow-Rankine: f = 40.000
Bouscaren-Rankine
Cooper-American Bridge Co. columns f = 30.000
Cooper-Phoenix columns f = 36.000
Cooper-Box & Channel columns f = 33.000
Bouscaren-Rankine: Keystone & American columns f = 36.000
Rankine, Round ends f = 36.000
Breaking loads in pounds per square inch.
Values of length divided by least radius of gyration.
42.000
40.000
38.000
36.000
34.000
32.000
30.000
28.000
26.000
24.000
22.000
20.000
18.000
16.000
14.000
12.000
10.000
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
210
220
230
240

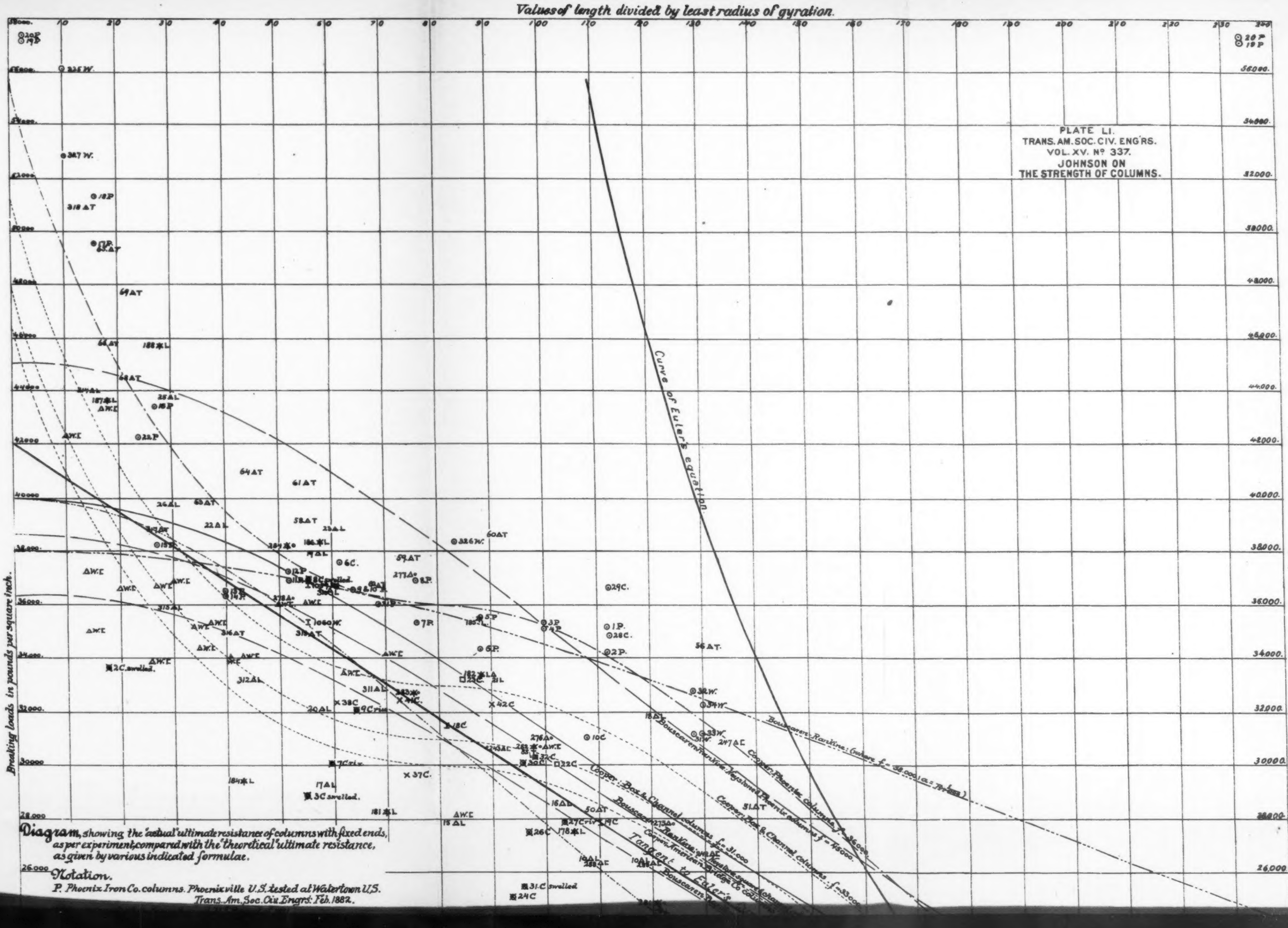
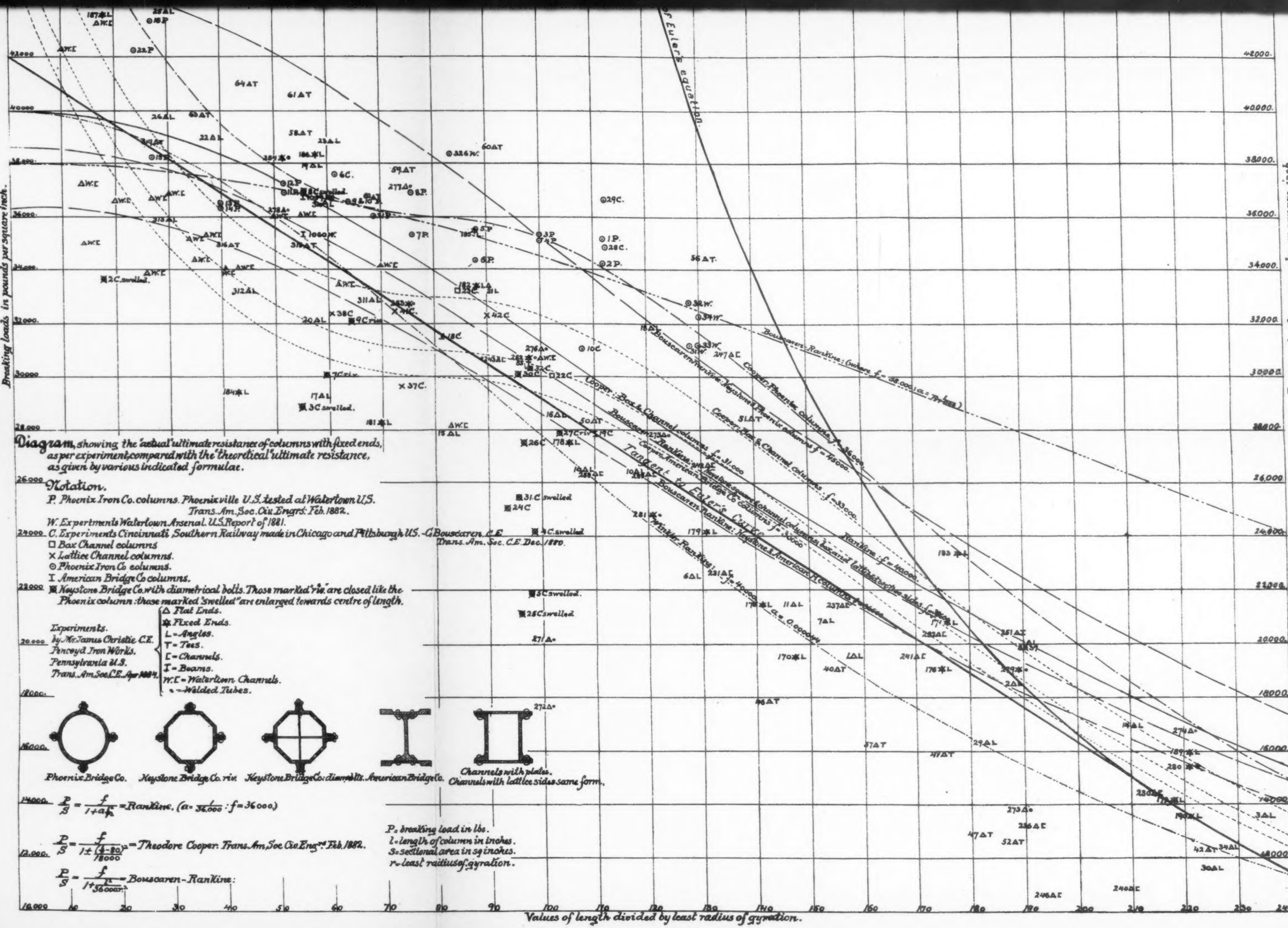


PLATE LI.
TRANS. AM. SOC. CIV. ENG'RS.
VOL. XV. Nº 337.
JOHNSON ON
THE STRENGTH OF COLUMNS.

Diagram, showing the "actual" ultimate resistance of columns with fixed ends, as per experiment, compared with the "theoretical" ultimate resistance, as given by various indicated formulae.

Notation.

P. Phœnix Iron Co. columns. Phœnixville U.S. tested at Watertown U.S. Trans. Am. Soc. Civ. Engrs. Feb. 1882.


Diagram, showing the "actual" ultimate resistance of columns with fixed ends, as per experiment, compared with the "theoretical" ultimate resistance, as given by various indicated formulae.
Notation.
P. Phoenix Iron Co. columns. Phoenixville U.S. tested at Watertown U.S. Trans. Am. Soc. Civ. Engrs: Feb. 1882.
W. Experiments Watertown Arsenal. U.S. Report of 1881.
C. Experiments Cincinnati Southern Railway made in Chicago and Pittsburgh U.S. - G. Bouscaren, C.E. Trans. Am. Soc. C.E. Dec. 1880.
□ Box Channel columns
× Lattice Channel columns.
⊙ Phoenix Iron Co. columns.
I American Bridge Co. columns.
Keystone Bridge Co. with diametrical bolts. Those marked "riv." are closed like the Phoenix column: those marked "swelled" are enlarged towards centre of length.
Experiments. by Mr. James Christie C.E. Pencoyd Iron Works. Pennsylvania U.S. Trans. Am. Soc. C.E. Apr. 1884.
△ Flat Ends.
Fixed Ends.
L = Angles.
T = Tees.
C = Channels.
I = Beams.
W.C = Watertown Channels.
Welded Tubes.
Phoenix Bridge Co.
Keystone Bridge Co. riv.
Keystone Bridge Co: diam. bolts.
American Bridge Co.
Channels with plates. Channels with lattice sides same form.
$\frac{P}{S} = \frac{f}{1 + a\frac{l^2}{r^2}}$ = Rankine. ($a = \frac{1}{36000}$: $f = 36000$)
$\frac{P}{S} = \frac{f}{1 \pm \frac{(\frac{l}{r} - 80)^2}{18000}}$ = Theodore Cooper. Trans. Am. Soc. Civ. Engrs. Feb. 1882.
$\frac{P}{S} = \frac{f}{1 + \frac{l^2}{36000 r^2}}$ = Bouscaren-Rankine:
P = breaking load in lbs.
l = length of column in inches.
S = sectional area in sq. inches.
r = least radius of gyration.
of Euler's equation.
Tangent to Euler's Curve.
Bouscaren-Rankine: Keystone & Phoenix columns.
Cooper: Phoenix columns f = 36000.
Cooper: Box & Channel columns f = 31000.
Cooper: Box & Channel columns f = 33000.
Cooper: American Bridge Co. columns f = 30000.
Rankine: f = 40000.
Bouscaren-Rankine: Keystone & American columns.
Bouscaren-Rankine: Gukes f = 36000. a = 1/25000
Breaking loads in pounds per square inch.
Values of length divided by least radius of gyration.

AMERICAN SOCIETY OF CIVIL ENGINEERS.
INSTITUTED 1852.

TRANSACTIONS.

NOTE.—This Society is not responsible, as a body, for the facts and opinions advanced in any of its publications.

338.
(Vol. XV.—July, 1886.)

NEW FORMULA FOR COMPRESSION MEMBERS.

BY PROFESSOR R. KROHN.
PRESENTED APRIL 7TH, 1886.

The determination of the permissible strains upon compression members has not yet been rationally and satisfactorily solved. Neither theoretical deductions nor practical tests have given us formulas free from objections.

Upon the assumption that the compressive force acts exactly in the direction of the axis of the member, theory teaches us that a bending or deflection of the member, and consequently the danger of crippling, can only occur when the external force has reached a certain limit, and as long as the force remains below this limit the member will only be strained by simple compression. Such a result, however, does not agree with the results of practical experiments, which rather show a deflection of the struts at almost any applied strain, and rupture under loads which, by theory, should not even produce deflection. This contradiction of the theory and the results of experiments, shows that the assumption of the theory that the external forces act through the axis of the mem-

bers, is untenable in practice. It is impossible, with the greatest care, to attain this condition; the unavoidable inaccuracies of workmanship and inequalities of the material render it impossible, even in carefully prepared test pieces, to secure this theoretical condition; how much more so then in practical bridge members?

Why, not, then, from the beginning, assume that the external forces do not coincide with the axis of the struts, but that the stresses are eccentric to this axis? The resultant formulas will then contain a value for the original eccentricity. Is it possible to determine this value from the results of experiments?

It is evident that the original eccentricity cannot be confined to any definite law. The results of experiments show great irregularities and wide variations. It is only possible then to determine and rely upon the upper limit of this original eccentricity for the ordinary conditions of practice. This determination would be a difficult task. We could not use the usual compression tests extending to the crippling point, but only such tests as determine the deflections of the struts while the fiber strains are within the elastic limit. But even this method would give us no reliable results, as the fiber strains would depend so much upon the accuracy of the workmanship and the carefulness with which the end connections are made. If an upper limit for this eccentricity were obtained from a particular series of experiments, we have no surety that it would apply to any other series of struts used in any particular structure.

While the foregoing theoretical and practical methods seem unsatisfactory, it would appear that we may take advantage of the following practical consideration to reach a solution.

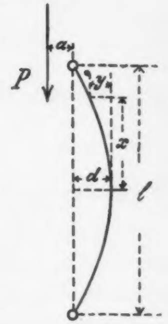
The tension and compression members used in our structures are made with equal care in the selection and arrangement of the component parts and in manufacturing the same. For similar conditions of practice, the original eccentricity of the lines of stress occurs in tension as well as in compression members, whether due to want of uniformity of the material or to inaccuracies in centering the connections. The additional strains produced by this condition of eccentricity are naturally smaller in the tension than in the compression members. Our experience teaches us that the original eccentricity is generally so small that the additional strains produced by it in the tension members may be neglected.

If we compare then the action of an external force upon a tension and a compression member having the same eccentricity, it will be possible to determine an upper limit for the strains in the compression members with the same relative precision and accuracy with which the strains in the tension members can be calculated.

Let Figure 1 represent a strut with its ends secured in such a manner that it is free to change the inclination of its axis at the ends.

Call its length l.

Let a force, P, be applied at a distance a from the axis of the strut.



$S =$ sectional area of strut.

$I =$ its moment of inertia.

$r =$ least radius of gyration.

$e =$ distance of extreme fiber from the neutral axis

$d =$ deflection of the elastic line.

$E =$ modulus of elasticity of the material.

x and $y =$ co-ordinates of deformed axis.

Then for any point in the deformed axis we have the following equation of moments:

$$M = P\ [\ a + d - y]$$

and for the equation of the elastic line:

$$\frac{d^2\, y}{d\, x^2} = \frac{P}{E\, I}\,[a + d - y]$$

Integrating this last equation twice, considering that for $x = 0$, $y = 0$ and $\frac{d\, y}{d\, x} = 0$ also, we get the following:

$$\frac{y}{a + d} = 1 - \cos. \left[\, x \sqrt{\frac{P}{E\, I}}\,\right]$$

If $x = \frac{l}{2}$, $y = d$, and we get:

$$\frac{d}{a + d} = 1 - \cos. \left[\frac{l}{2} \sqrt{\frac{P}{E\, I}}\,\right] \text{or}$$

$$a + d = \frac{a}{\cos. \left[\frac{l}{2} \sqrt{\frac{P}{E\, I}}\,\right]} \tag{1}$$

The greatest compression in the extreme fiber will be at the middle of the strut. Denote this by C:

$$C = \frac{P}{S}\left[1 + \frac{(a+d)\,e}{r^2}\right] \tag{2}$$

Combining equations (1) and (2):

$$C = \frac{P}{S}\left[1 + \frac{a\,e}{r^2 \cos.\left[\frac{l}{2}\sqrt{\frac{P}{E\,I}}\right]}\right] \tag{3}$$

The cosine in this formula may be developed into a progression, and by remembering that $\frac{l}{2}\sqrt{\frac{P}{E\,I}}$ will be smaller than unity,* we can obtain a very close approximation by taking

$$\cos.\left[\frac{l}{2}\sqrt{\frac{P}{E\,I}}\right] = 1 - \frac{1}{8}\,\frac{l^2\,P}{E\,I}$$

Substituting this value in equation (3), we get:

$$C = \frac{P}{S}\left[1 + \frac{a\,e}{r^2\left(1 - \frac{1}{8}\,\frac{l^2\,P}{E\,I}\right)}\right] \tag{4}$$

If a similar member were strained in tension by the force P acting at the same eccentricity a, the bending moment at no point of the axis would be greater than $P\,a$; and the greatest tension fiber strain would be

$$T = \frac{P}{S}\left[1 + \frac{a\,e}{r^2}\right] \tag{5}$$

Equations (4) and (5) then represent the relative maximum strains produced on similar compression and tension members by similar forces.

If we call $\frac{C}{T} = n$, n denotes the ratio between the induced strains, or how many times greater the maximum compression is than the maximum tension under similar conditions in practice.

---

*If the original eccentricity equals zero, we have for the load producing rupture $\pi^2\,\frac{E\,I}{l^2}$. As, however, there is always some eccentricity, we can assert from both theory and practice that the load R, producing rupture, will be

$$R < \pi^2\,\frac{E\,I}{l^2} \quad \text{or} \quad \frac{l}{2}\sqrt{\frac{R}{E\,I}} < \frac{\pi}{2}$$

If the strains are kept within the highest allowable limit, the load P must be far less than R; we can therefore for all practical cases assert that

$$\frac{l}{2}\sqrt{\frac{P}{E\,I}} < 1.$$

From (4) and (5) we get:

$$n = 1 + \frac{\frac{1}{8}\frac{l^2 P}{E I}}{1 - \frac{1}{8}\frac{l^2 P}{E I}} \times \frac{\frac{a\,e}{r^2}}{1 + \frac{a\,e}{r^2}} \tag{6}$$

The fraction $\frac{\frac{a\,e}{r^2}}{1 + \frac{a\,e}{r^2}}$ can never be greater than unity, whatever value the original eccentricity a may have. We have therefore for the upper limit of the value of n, the following equation:

$$n = \frac{1}{1 - \frac{1}{8}\frac{l^2 P}{E I}} \tag{7}$$

By maintaining this ratio between our working strains in compression and tension, we can neglect equally well the eccentricity for compression members as in practice we do for tension members.

Calling $k = \frac{P}{S}$ the allowed unit strain in tension members, the allowed unit strain in compression members will be

$$\frac{P}{S} = \frac{k}{n} \tag{8}$$

From (7) and (8) we get for compression

$$\frac{P}{S} = k\left[1 - \frac{1}{8}\frac{l^2 P}{E I}\right]$$

Substituting for I its value $S\,r^2$, we get:

$$\frac{P}{S} = \frac{k}{1 + \frac{1}{8}\frac{k}{E}\frac{l^2}{r^2}} \tag{9}$$

which agrees in its general form with Rankine's formula; the coefficient for $\frac{l^2}{r^2}$ however is not constant, but varies with $\frac{k}{E}$; that is, it increases with increased deformation of the material.

For example: With wrought-iron, calling $k = 10\,000$ pounds per square inch, and $E = 26\,000\,000$ pounds, the comparative working strain for compression members should be

$$\frac{P}{S} = \frac{10\,000}{1 + \frac{1}{20\,800}\,\frac{l^2}{r^2}}$$

For steel, with the same modulus of elasticity, but using a higher unit strain for tension, viz., $k = 15\,000$, we get:

$$\frac{P}{S} = \frac{15\,000}{1 + \frac{1}{13\,800}\,\frac{l^2}{r^2}}$$

which shows that for long and slender columns, whose ratios for $\frac{l^2}{r^2}$ are large, steel cannot be used with the same advantage as in short, massive columns, or as in tension members.

It will be necessary, in order to use steel economically for compression members, to employ such shapes as will give a large radius of gyration in proportion to the length of the struts.

For cast-iron:

$$k = 10\,000 \quad E = 13\,000\,000$$

$$\frac{P}{S} = \frac{10\,000}{1 + \frac{1}{10\,400}\,\frac{l^2}{r^2}}$$

For wood:

$$k = 1\,000 \quad E = 1\,200\,000$$

$$\frac{P}{S} = \frac{1\,000}{1 + \frac{1}{9\,600}\,\frac{l^2}{r^2}}$$